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The provision of local public goods and demographic change

Thesis submitted in partial fulfillment of the requirements for the degree of

Dr.rer.pol.

by

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Abstract

The main contribution of this thesis is a comprehensive analysis of the influence of changes in the population structure on local communities, in particular with respect to the provision of publicly provided goods. The focus is placed on the consequences of two of the major processes of demographic change, namely aging and shrinking. The three main chapters of this contribution consider the effects at the local level from both a theoretical and an empirical perspective. The first model focuses on the influence of population aging on the provision of local publicly provided goods, when the young population may relocate. When aging advances, gerontocracies and social planners substitute publicly provided goods aimed at the mobile young for publicly provided goods for the elderly. However, due to fiscal competition, gerontocracies will provide even more of the publicly provided good for the young than the social planner. The second model considers in a two-period setting, the interaction of a shrinking population when the investments made by the previous generation are long lived. The laissez-faire and welfare maximizing outcomes are computed for two cases; first with no costs of upkeep and second for the case when costs of upkeep accrue. A comparison of the solutions shows that public provision for the first generation is inefficiently low in laissez-faire when there are no costs of upkeep. However, if costs of upkeep accrue, the laissez-faire outcome for the intergenerational publicly provided good may be too high. Chapter four contains an empirical analysis. In a two-stage analysis the efficiency of the provision of child care services in municipalities is evaluated in the German State of Saxony. First, the results of the Data Envelopment Analysis (DEA) show substantial efficiency differences; the median municipality is up to 28% inefficient. In a second stage bootstrapped truncated regression, determinants of the inefficiency are identified. Explanatory variables such as an uncompensated mayor or a larger share of over 65-year-olds significantly increase inefficiency.

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Chapter 1

Introduction

The economic consequences of a changing population structure are wide reaching. Yet demographic processes are relatively predictable and slow, giving governments and individuals time to adapt. Adaptation, both private as well as government induced, can help mitigate any potentially adverse consequences for society. For example: investment in education of the labor force, investment in child care to increase the labor force participation of women, preventative health care to allow older workers to remain active longer, more flexible nursing home solutions etc. may reduce the burden on future generations. This contribution focuses on the consequences of demographic changes at the local level. Even if ageing and shrinking of the population are often documented at the national level, the ongoing population processes differ substantially on a smaller spatial scale. Comparatively very little economic research has been conducted on this specific topic. The underlying question therefore is: how does demographic change influence local communities? The main contribution of this thesis is a detailed analysis of the influence of changes in the population structure on local communities; in particular with respect to their ability to produce the local public services demanded by their populations.

In the past the impacts of demographic change have received broad attention, from (economic) researchers in nearly all countries that are/will be affected. As the population pyramid becomes increasingly inverted the sustainability and reform of public pay-as-you-go pension schemes is one major concern (Bongaarts, 2004; Fehr, 2000). Others have considered the impact of aging on savings behavior (Bloom *et al.*, 2007) and capital markets (Krueger and Ludwig, 2007; Poterba, 2001, 2004; Börsch-Supan and Ludwig, 2009); a topic of renewed interest in light of the recent financial crisis in particular with respect to losses in private pension savings (Whitehouse, 2009). Other research considers the demographic prospects with respect to the labor market (McDonald and Kippen, 2001). Here two main factors are considered, namely the labor force participation and the (age specific) productivity of the workforce. Studies have shown that an old labor force is not necessarily less productive and that a mixed age structure is important for productivity (Skirbekk, 2008; Prskawetz *et al.*, 2008). In many countries additional labor market reforms may still be necessary to remove incentives for early retirement, which would in turn also reduce the burden on the pension systems (D'Addio *et al.*, 2010). In addition to the vast issue of pension reform, other aspects of fiscal policy have also received increased attention in light of the demographic developments. The increasing costs of health care (including the growing burden of long-term nursing care) are becoming ever more urgent (Breyer *et al.*, 2010; Comas-Herrera *et al.*, 2006; Cutler and Sheiner, 2001).

This introduction presents the general framework in which the ensuing main Chapters 2, 3 and 4 are embedded. Therefore, the trends of the demographic transition are briefly presented in the following section. The projected population developments in many European countries are similar in terms of low birth rates and increasing life-expectancy. Although institutional settings may be different, common consequences are expected, and therefore also more general conclusions can be derived from the ensuing analyses. In particular with respect to the models presented, the results are largely independent of national contexts.

In Germany public discussions on *"demographischer Wandel"* (demographic change) regularly reoccur in different domains. Moreover, the eastern German states are particularly affected by population ageing and comprise one of the first areas to experience large scale population loss. The data for the empirical analysis in the last chapter reflects this, but also the theoretical models in the other two main chapters were also conceived with the German setting in mind. Thus the German institutional framework is also briefly introduced. Additionally important economic concepts that recur in the ensuing chapters are also presented in this brief introduction. Finally, the contributions of each chapter are highlighted.

1.1 Demographic trends

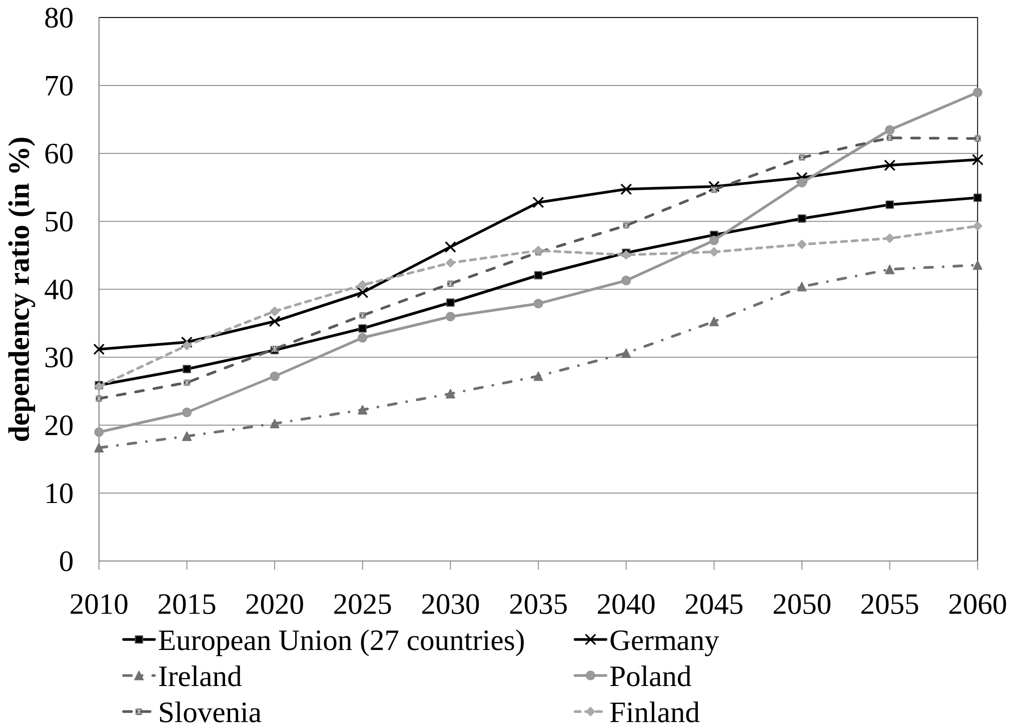
Due to high birth rates in developing countries, the global population is growing rapidly. However, many developed countries that have gone through the second demographic transition are now faced with have stagnating and even shrinking populations (see for instance: Van De Kaa (1987) and Lesthaeghe (2010)).

The population size of a region in a given time is determined by two factors: natural population change and migration. The natural population size is determined by the birth and mortality rates. Birth rates are in turn highly dependent on the number of women of reproductive age as well as the fertility rate. Whereas mortality is influenced by the age structure of the population. Moreover, migration can either contribute to population growth or decline. In particular immigration of young persons, especially women, can thus have a positive impact on natural population change. On the one hand, the relative mortality is lowered by migration of relatively young persons. On the other hand, the immigrant women may contribute to the birth rate by having children, which further shifts the age structure. Conversely, the emigration of young can reduce the natural population growth rates in the region of origin.

1.1.1 Population aging through increasing longevity

With total fertility rates persistently below the replacement rate ($TFR < 2.1$) in European countries, population loss is inevitable if the natural deficit is not compensated by international immigrants. Simultaneously, people are living longer. Together low birth rates and increased longevity lead to an increasing old-age-dependency ratio (the number of over 65 year olds divided by the population aged 15-64 years times 100). This indicator of the burden placed on the working age population is depicted in Figure 1.1 for selected European countries as well as the average for the European Union (EU with 27 member states). Clearly the dependency ratios are projected to increase in all of the countries, although at different rates. The fastest change is in Poland, where the ratio is projected to increase to the highest level in the EU with almost 70% in 2060 from less than 20% in 2010. An important contributing factor to this marked increase is the persistently high rate of emigration of young Poles. Conversely, in Ireland the increase is relatively slow (from about 17% in 2010 to 43% in 2060).

Figure 1.1: Projected old age dependency ratios (number of over 65 year olds as a percentage of 15-64 year olds) in selected EU countries, 2010-2060



Source: Eurostat (2011)

Clearly such large demographic shifts have wide reaching impacts on many aspects of life in the affected societies. While demographic processes pose challenges to policy makers at all levels of government, this contribution concentrates on the local government provision of services. Furthermore, the focus is placed on the consequences of two of the major process of demographic change, namely aging and shrinking. Thus the ongoing demographic developments frame the topic. The consequences of potential generational conflicts and the inefficiency that may arise from changes in the municipal population structure are considered. This contribution, however, does not aim to suggest or evaluate population policy.

The trend in Figure 1.1 depicts the significant projected population ageing in European countries, at the national level. However, the developments at the subnational levels are more differentiated. In Germany the largest agglomerations tend to have younger populations, while the countryside is becoming ever less populated (see Table 1.1)[1]. Since the young tend to be more mobile, this implies that ageing in the rural areas is also progressing more rapidly. In particular the future population growth in Germany is expected to take place in the sprawl areas of the largest cities (Berlin, Munich and Hamburg). Moreover, vast areas of eastern Germany will experience population losses. The contrasts are also sharp, in fact some counties (German *Landkreise* and *kreisfreie Städte*) experiencing growth are located in the immediate geographic vicinity of counties that will lose population.

Precisely due to the option to relocate, the effects of population ageing and shrinking are distinctly different at the sub-national levels.[2] Chapter 2 of this contribution deals in particular with the possibility of reloca-

[1] The western German states refer to the ten states in addition to Berlin that formed the Federal Republic of Germany before the reunification in 1990, namely: Baden-Württemberg, Bavaria *(Bayern)*, Bremen, Hamburg, Hesse *(Hessen)*, Lower Saxony *(Niedersachsen)*, North Rhine-Westphalia *(Nordrhein-Westfalen)* Rhineland-Palatinate *(Rheinland-Pfalz)*, Saarland and Schleswig-Holstein. In turn the eastern German states refer to those that comprised the former German Democratic Republic (GDR) and include; Brandenburg, Mecklenburg-Western Pomerania *(Mecklenburg-Vorpommern)*, Saxony *(Sachsen)*, Saxony-Anhalt *(Sachsen-Anhalt)* and Thuringia *(Thüringen)*.

[2] Spatial mobility is of course also possible at the national scale, however, both the monetary and emotional costs tend to increase with increasing distance.

Table 1.1: Share of old and aging in different spatial types in the eastern and western Germany

Region type	**Share of over 65-year-olds in 2008**		**Change in over 65-year-olds 2003-2008 (in %)**	
	west	east	west	east
Urban areas	19.7	20.4	8.4	18.8
Densely populated non-urban areas	19.9	24.4	11.7	14.6
Rural areas in vicinity of urban areas	20.1	22.5	11.0	16.9
Rural areas	20.4	23.0	13.6	12.0

Source: Federal Institute for Research on Building, Urban Affairs and Spatial Development (BBSR) (2010)

tion. Whereas democratic voting determines the fiscal policy at the national level, at the regional and local levels the possibility to "vote with one's feet" (Tiebout, 1956) in the pursuit of a more suitable bundle of publicly provided goods presents an additional option for a mobile constituency. At the local level, fiscal competition between local governments may therefore take place over the bundle of publicly provided goods. Such competition may become more intense as the size of the population shrinks or the young segment becomes more scarce.

Furthermore, where population loss is rapid, the consequence might be over-sized infrastructure: too many school buildings, idle sewage capacity for many more households, and unused sports arenas. When decision makers are unaware of the local population developments, misinvestments are likely. Even if no malice is intended it is probable that in the future, communities will be faced with a stock of infrastructure intended for a larger population. The potential burdens that this may place on the smaller future generations is considered in Chapter 3.

1.1.2 Low fertility rates

When the population ages the supply of qualified labor may become scare and hence the activation of labor force reserves will increase in importance. Particularly increasing the labor market participation of women and mothers is a source of qualified labor. The burden of the dual role of women as mothers and full-time employees may also contribute to consistently low fertility (Ahn and Mira, 2002; Björklund, 2006; Brewster and Rindfuss, 2000; Del Boca, 2002; Hank and Kreyenfeld, 2003; Vos, 2009). Potentially a lack of high-quality child care is also a contributing factor. The main policy instrument for the state is then to offer such high-quality care to encourage working women to also bear children. This also reduces the risk of employers to hire women. In Germany a relatively small share of women, participate full-time in the labor force. As Table 1.2 shows, the labor force participation rate of females in the selected countries are similar, however with almost 40% the prevalence of part-time employment is much higher in Germany and the United Kingdom (UK).

Table 1.2: Female labor force participation in selected countries

Country	Female labor force participation[a]	Female part-time employment[b]
Germany	71.0	38.1
France	66.1	22.4
Denmark	76.7	24.8
Finland	74.1	15.9
UK	70.4	38.8
EU-27	65.3	n.a.

[a] Civilian labor force participation of females as % of female population aged 15-64 years
[b] % of part-time employed females of total female civilian employed

Source: Annual Labour Force Statistics OECD (2011).

Through the expansion of the supply of child care, especially for children under the age of three, the state may hope to motivate more highly qualified

Table 1.3: Proportion of children cared for in formal arrangements in selected countries (in % of children in the age group)

Country	Children up to 3 years old	Children from 3 years to school age
Germany	18	93
France	31	94
Denmark	73	96
Finland	26	77
UK	33	89
EU-25	26	84

Source: European Commission (2008)

women to have more children while not relinquishing their position in the work force. The organization of child care is thus an important issue.

In most countries public child care is provided by the local government. In Germany, the acceptance, use and supply of out of home care for young children is still marked by an east-west divide. While in the former German Democratic Republic (GDR) most mothers worked, and a comprehensive system is still in place in the eastern states, the participation rates in the western states are much lower. In eastern Germany 48% of children aged 0-3 attend child care, whereas the corresponding value for western Germany is merely 17% (Federal Statistical Office, 2011b). The efficient provision of this local service remains important. As the municipal budgets are getting ever tighter, the use of the resources is ever more scrutinized. Chapter 4 of this contribution evaluates the efficiency of the use of resources in the provision of child care in the municipalities in the eastern German state Saxony. Furthermore it shows that demographic factors play a decisive role in the spending on child care.

1.2 Key economic terms and concepts

The demographic reality presented in the previous section forms one basis for the analyses in the three main chapters of this thesis. The other corner stone for the analysis is the local provision of public goods. In the following sections some concepts that form the basis for the analyses in the main chapters are introduced. Under the heading "the role of local government in the public sector" the topics included are: local public goods, fiscal federalism and competition and intergenerational public goods. An additional section is dedicated to briefly explain local public finance in Germany. Moreover, the references more closely tied to the models and ideas in a given chapter are contained therein.

1.2.1 The role of local government in the public sector

Local public goods

Local (e.g. impure) public goods are characterized either as partially rival and/or excludable (Sandler and Tschirhart, 1980). The formal definition of local public goods stems from the theory of pure public goods (Samuelson, 1954). As opposed to pure public goods (such as national defense), the benefits of impure public goods may decline with every additional user (partial rivalry). The benefits of a local public good are generally defined by a limited spatial scale (parks). Conversely, local public goods may also induce benefit spillovers if the exclusion of non-residents is not possible.[3] The works by Buchanan (1965) and Tiebout (1956) on clubs and the revelation of preferences for publicly provided goods make the foundation for the analysis of local provision of public goods.

Buchanan proposed a general theory of "clubs", where a group shares an impure public good characterized by congestion and excludable bene-

[3] Spillovers from local public goods may be either positive or negative. If for instance one local community provides education, and then members who have received this education emigrate, it entails a positive spillover onto the community to which they move. Pollution emitted in one community that also adversely affects its neighbor is an example of a negative spillover.

fits. Club goods are excludable since each member's admission depends on her willingness to pay. Ultimately both the Tiebout and Buchanan models rely on the notion of an optimal group size arising from the trade-off between benefits from sharing and disutility from crowding. According to Buchanan's Theory of Clubs, optimal provision of local public goods in an economy can be achieved by establishing demand-homogeneous communities. In lieu of this restrictive implication, the analysis of mixed communities is achieved by assuming complementarities between different types of individuals.[4] Efficiency achieved through such homogeneous communities hinges crucially on the underlying assumption of free mobility, which is also implied by Tiebout.

In such a setting where individuals are partitioned into clubs with homogeneous members, a decentralized mechanism can achieve Pareto optimality in local public goods according to the Tiebout hypothesis (Cornes and Sandler, 1996, p. 352). The preference revelation problem inherent to public goods is overcome through the act of "voting with one's feet." Additional assumptions include: perfect information, mobile voters (not restricted by employment opportunities), many municipalities and no benefit or tax spillovers. Under these assumptions individuals are able to choose the community which offers their most desired bundle of public goods, thus achieving an efficient level of provision in stratified communities.

Since these theoretical considerations, volumes of empirical studies have been published. In the United States and Canada, many studies follow the tradition initiated by Borcherding and Deacon (1972) and Bergstrom and Goodman (1973) in estimating the demand for public goods at a sub-national level. Characteristically these studies employ a median voter model to estimate demand for specific categories of local public goods in terms of price and income elasticities. These studies have verified the non-existence of pure public goods at the local level. Instead local public goods are impure and yield very little economies of scale in consumption (Reiter and Weichenrieder, 1997).

4 See Berglas (1976a) and Berglas (1976b), Brueckner (1979) and Brueckner (1994), McGuire (1972), McGuire (1974) and McGuire (1991) and Sandler and Tschirhart (1980) for a survey. The above references are not intended to represent an exhausting listing of literature on club theory.

In the above discussion the public goods are defined over the benefit they award. Implicitly it is thus assumed that each type of good can be supplied by a special district of optimal size. This is of course unrealistic, since in reality communities instead supply a bundle of different services. Instead the size of a city is also determined by complementarities in the production of these services and by negative effects of agglomeration. Fiscal federalism can be seen as a compromise resulting from the inefficiency of infinite special districts. A limited number of different tiers of government are in charge of providing the types of goods they can provide most efficiently.

The idea of local public goods is essential to all ensuing chapters. In particular, in Chapters 2 and 3, a specific relation between the benefit each individual derives from the provision of public goods to the local population size is assumed. Child care, a specific local public good, is considered in Chapter 4.

Fiscal federalism and fiscal decentralization

Fiscal federalism describes the workings of the public sector in a multi-tiered government framework. From a public finance perspective the need for a multi-tiered system arises, given the characteristics of different public goods and services. The functions that different levels of government should perform (Musgrave, 1959) and the "decentralization theorem" (Oates, 1972) have given rise to a broad literature on fiscal federalism and public finance. Fiscal decentralization takes place in two dimensions, when the power to tax is given to a sub-national level or a sub-national level has the responsibility for implementing an expenditure function. In recent years, many developing countries have undergone fiscal decentralization.

Musgrave divides the fiscal functions into three different categories: allocation, distribution and stabilization. Accordingly a fiscal federal system can be arranged so that each level performs those functions it is most suited for. The central level is usually responsible for the stabilization of fiscal and monetary operations as well as redistribution functions, whereas the responsibly for allocation should rest with both local and central government

(Oates, 1968). Similar considerations apply to which tax instruments should be available to the different levels of government to finance these services. Arguably the most mobile tax base (personal income and corporate taxes) should be taxed by the highest level, while the local level should either rely on user charges, or on relatively immobile tax bases such as the property tax. Since incomes are likely unequally distributed within the federation, either discriminatory taxation or inter community transfers could be used by the central government. Through such transfers between communities (subsidies) the central government could induce welfare maximizing levels of public goods provision in all sub-national jurisdictions (Oates, 2005). The "decentralization theorem" further presupposes that it would be politically unfeasible for a central government to provide different levels of public goods in jurisdictions of the same tier.

Accordingly the allocation function should be maintained by the level able to most economically provide the good or service. Generally this will be as close to the consumer as possible. Therefore, the local level is justified in performing certain functions, which are at least as cost efficiently provided at the local level as they would be at the state or central levels. Since the consumption of publicly provided goods is confined to a geographic subset of the population, the mobility of agents is not explicitly considered.

Justification for local public goods provision is based on the more detailed knowledge and information available at the local level, as well as the diversity of tastes across larger areas. Furthermore, competition among the many jurisdictions may lead to more efficient outcomes, while also allowing for more experimentation and innovation at a lower risk. If a local policy is deemed good, then neighboring jurisdictions will imitate it. Furthermore, through the competition that arises between jurisdictions it is possible from a public choice perspective, that decentralization could prevent excessive growth of the public sector (Brennan and Buchanan, 1980, p. 168-186).

In particular in Chapter 2 the federal structure is implied since the municipalities receive per capita transfers to finance their chosen level of service provision. Furthermore, the provision is considered in a framework of multiple communities. The inter-municipal competition aspect is further elucidated in the next section. In Chapter 4 the federal structure in Germany is reflected in the empirical analysis. The sample consists of municipalities from one state (the Free State of Saxony). In particular the effect of the share of revenues from inter-governmental grants of municipal revenues on the efficiency on the provision of child care is tested. It is found that efficiency in child care provision is independent of the size of grants a municipality receives.

Fiscal competition

Besides the potentially higher efficiency with which public provision can be maintained by sub-national jurisdictions, the competition between them may be efficiency enhancing. Whether competition is considered vertical or horizontal depends on the actors involved. Vertical refers to the competition between different tiers of government that tax the same activity and thus share the same tax base. Whereas horizontal refers to competition between jurisdictions of the same sub-national level.

Broadly speaking, fiscal competition deals with the externality imposed by a mobile productive resource on a jurisdiction (Wildasin, 2003a).[5] The literature within this scope is extensive, but can generally be divided into three sub-categories according to the arena in which jurisdictions compete. First, tax competition may be considered, where jurisdictions seek to offer the lowest tax price while still providing comparable services. Secondly, competition may arise with respect to different aspects of the public goods bundle offered, including quality, cost and composition. Finally, non-tax instruments (excluding expenditures) may be used to attract investment in local businesses, in order to increase production, employment and income and thus broadening the local tax base (Wilson, 1999).

A distinction with respect to the mobility of resources is fundamental in fiscal competition. Generally different affinities can be postulated for labor

[5] Technically, welfare competition can also arise without mobility through yard-stick competition (Besley and Case, 1995), where residents are able to receive information from neighboring communities. The fiscal externality literature, including fiscal competition, expenditure competition, welfare competition, and tax competition already covers a wide range of issues.

and capital, or finer distinctions can be made with respect to skill levels, and age of labor, or sector and type of capital.[6] In analyzing the composition of public spending, Keen and Marchand (1997) show that not only will tax competition drive taxes to inefficiently low levels, but that the spending also becomes skewed in favor of the mobile factor.[7]

In Chapter 2, competition is distinguished as taking place over the level of public goods provision. There is heterogeneity between population groups as the young individuals are mobile whereas the elderly are immobile. The influence of competition is also considered in Chapter 4, by taking account of facility density within a municipality in the efficiency analysis and considering the influence of the presence of non-public child care facilities on the efficiency of public facilities. However, inter-jurisdictional competition is not directly analyzed.

Intergenerational public goods

Government debt incurred today may not be repaid for many years, potentially even generations from now. Thus debt is an intergenerational issue. Other examples include: pay-as-you-go pension systems, investments in long lived public goods and environmental protection (Sandler, 1999). Similarly, the idea of an intergenerational or a durable public good stems from the fact that some decisions and investments have long lasting impacts. Intergenerational public goods may have the characteristics of an impure public good. Like local public goods intergenerational public goods may be rival in consumption (and may thus be congested by the use) within a period. Furthermore the benefit of an intergenerational public good may be influenced

[6] For instance Keen and Marchand (1997); Borck (2005) consider high and low skilled workers. Welfare competition in general deals with mobile poor and immobile wealthy (Brueckner, 2000). Wildasin (2003b) considers endogenously determined mobility rates of labor and capital.

[7] The same conclusion is reached with respect to public expenditures in favor of the mobile factor when agents are heterogeneous (Borck, 2005). When skilled workers are mobile and a capital skill complementarity exists, public expenditures geared toward the mobile skilled labor are relatively higher than services for the unskilled and immobile labor.The theoretical model developed in Borck (2005) is used to empirically verify fiscal competition at the local level in Germany in Borck, Caliendo and Steiner (2006).

not only by current use, but the current use may also influence (reduce) future benefits. Whereas fiscal competition over local public goods deals with the spillovers within a given period between jurisdictions, intergenerational public goods induce spillovers between two or more periods.

Since the unborn future generations are unable to vote or sign contracts with their predecessors, they may inherit a suboptimal stock of publicly provided goods or be subjected to debt repayments. Nonetheless, land price capitalization and altruism represent two potential alternatives to reduce any adverse consequences for future generations. As discussed above, a fiscal federal structure can make the provision of local public goods more efficient. In a fiscal competition framework the capitalization of intergenerational spillovers into local land values can protect the future generations (Rangel, 2005). New residents are attracted to communities with low debt and high public goods provision which will in turn increase property values. Thus current residents have an incentive to vote for an attractive bundle of public goods and low debt, since they themselves will reap the benefits when selling their property. Intergenerational altruism may also remedy potentially negative spillovers from short-sighted decisions of current generations. If the members of the current generation derive some indirect welfare from the future consumption of their descendants, then long-term decisions will (at least partially) take the future benefit into account (Myles, 1997). Thus any adverse future consequences are reduced.

Chapter 3 considers an intergenerational publicly provided good that is affected by congestion. The benefit to each user is diluted by the number of users and the intergenerational inheritance of the stock of public good may benefit the subsequent generation. The laissez-faire and welfare maximizing outcomes are computed for two cases; first with no costs of upkeep and second for the case when costs of upkeep accrue. A comparison of the solutions shows that public provision for the first generation is inefficiently low in laissez-faire when there are no costs of upkeep. However, if costs of upkeep accrue, the laissez-faire outcome for the intergenerational publicly provided good may be too high.

1.2.2 Fiscal federalism and local public finance in Germany

In a fiscally decentralized economy, taxes can either be levied and allocated within a jurisdiction, or tax revenue from various levels can be pooled and subsequently redistributed vertically and horizontally. In Germany most of the revenues at the local level are not acquired from taxes raised locally, but rather through a complex system of grants and revenue sharing schemes between the other levels. The allocation of revenues between the states, and the local authorities follows the principle of equalization, according to which the provision of equal chances is to be guaranteed in all sub-regions. The Basic Constitutional Law (*Grundgesetz*) states that all regions should be able to perform their assigned functions to largely the same extent, independently of the local economic and fiscal means. This is achieved through both vertical (between different levels) and horizontal (between jurisdictions of the same level) equalizing grants (Nam *et al.*, 2001). The aim of this system is spatial and social cohesion in all regions of the country.

In Germany there are mainly three fiscally relevant levels of government: the federal (*Bund*), the state (*Länder*), and the local (*Gemeinde*) levels.[8] The Basic Constitutional Law guarantees the local level self-governance and administration, but the law also defines certain tasks to be performed by this level (in terms of public goods and services). In order to implement these functions, the local jurisdictions must engage in revenue sharing since their revenues from own sources are insufficient.

Over two-thirds of taxes raised in Germany are shared between the three levels. The two main sources are the personal income tax as well as value added tax (Krings, 2010).[9] Since the own sources of revenues are particularly small at the local level, the municipalities are highly dependent on the large transfers they receive from the other two levels. For instance in 2009, the municipalities (local level) raised about 40% of their current revenues from

[8] Additional administrative levels, include the districts *Regierungsbezirke* and counties (*Kreise*) both between the state and local levels in declining order.

[9] Of the total taxes levied (524 billion) in 2009, the personal income tax (135.2 billion €) and the value added tax (141.9 billion €) accounted for 26% and 27%, respectively.

taxes (including the shared taxes), and much of the remaining 60% was obtained from grants (Krings, 2010). The implementation is achieved by first dividing the tax revenue and the federal level compensating weak states (*Länderfinanzausgleich*).[10]

Thereafter within the municipal reallocation system (*kommunale Finanzausgleich*) the revenues of local authorities within a given state are redistributed with the aim of equalization. The size of local grants is state legislation, therefore at the national level no common means of determination of grant size is possible. However, generally the size of grants tend to be highly correlated from year to year and the revenue development at the local level is highly dependent on the revenues of the state. The constitution also does not imply a specific method of redistribution, it only requires that the distribution is fair. Generally the grant size is determined by the difference of the computed expenditure "need"[11] and the tax capacity of a municipality (Parsche and Steinherr, 1995; Steinherr and Parsche, 1998).

The reallocation system has been criticized for being too focused on population size as a determinant of expenditure need, and not accounting for several other important factors, including: age, employment, and economic structures, incidence of commuting, prevalence of environmental damage and hazards, age of local infrastructure. When a municipality is losing population relative to other municipalities, in a state with generally weak fiscal situation, then the local revenue losses will be significant. Such a situation currently prevails in many eastern German municipalities. Thus the main role of local level is to allocate the fixed resources between the services that the local authority is required to provide (Borge and Rattsø, 1995; Borge *et al.*, 1995). In Chapter 2 the allocation of grant revenues between two different types of publicly provided goods are modeled.

The efficiency of the use of resources on one particular type of publicly provided good, child care, is considered in Chapter 4. Furthermore, the influence of population loss is considered in a setting where the public goods provision is long lived.

[10] Refer to, for instance, Eltges (2006).
[11] The municipal expenditure "needs" remain to be subjectively determined.

1.3 The structure and contribution of this thesis

This work is intended to contribute to the discussion of the impact of the current demographic developments, in particular, its implications for local authorities. The theoretical models in Chapters 2 and 3 can be viewed in different national institutional arrangements, in particularly in federations with a relatively strong and independent local public sector. The distinctly German institutional setting is most important in the empirical analysis in Chapter 4.

1.3.1 The main contribution

The main contribution of this thesis is a comprehensive analysis of the influence of changes in the population structure on local communities; in particular with respect to the provision of publicly provided goods. This section briefly presents how the rest of this thesis is structured. Generally each ensuing chapter is a paper that I have worked on during my doctoral studies. The papers that are the basis for two of the three main chapters were written by a co-author and myself. Specifically, Chapters 2 and 4 were co-authored, where as Chapter 3 is not. In line with the university regulation, I will in the following describe my contribution to each paper in more detail.

1.3.2 Contributions to the chapters

Chapter 2 is based on the article "Ageing municipalities, gerontocracy and fiscal competition", which was published in the June 2010 issue of the *European Journal of Political Economy*, as well as on the working paper version of the same paper (*CESifo Working Paper Series* number *2469*).[12] Both versions are co-authored by my thesis supervisor Prof. Dr. Marcel Thum and myself. The initial input to begin work on my thesis, by modeling demographic change in a fiscal competition framework, came from my supervisor

12 See citations: Montén and Thum (2010) and Montén and Thum (2008).

and co-author. I proceeded to set up the model with two generations, two communities and two congestible (local) public goods. Thus the first draft of the paper is based on these ideas. Thereafter, we discussed the implications of the results in on multiple occasions. These conversations shaped the rest of the paper.

Chapter 3 is written entirely by myself. Although the contents have of course benefited from discussions with my supervisor, colleagues and the inputs of participants in the seminars in which I presented previous versions of the model.

Chapter 4 is also based on a cooperative work. The paper "Determinants of efficiency in child care provision", appeared as *ifo Working Paper Number 83* in March 2010, and is co-authored by my colleague Christian Thater at the Dresden Branch of the ifo Institute. This article has also recently been accepted for publication in a forthcoming issue of the journal *FinanzArchiv/ Public Finance Analysis.*[13] The original idea for the paper stems from a project we worked on together at the ifo Institute's Dresden Branch, with the title: "Making Saxon Municipalities Ready for Demographic Change – Policy Options". The contents of the paper and subsequently this chapter have changed substantially since the first drafts. Originally a mere efficiency analysis exercise was intended, but later the public finance implications of municipal spending on child care were brought to the forefront. For the working paper version of the paper, I computed the efficiency scores using the Data Envelopment Analysis (DEA) whereas my co-author implemented the regression analysis. In the course of the revisions for the final publication, I executed the regression analyses whereas my co-author was in charge of the efficiency analysis. The selection of the inputs and outputs (including the school readiness data) as well as the regression variables was done in common.

The ensuing three chapters can therefore also be treated as self-contained documents. However, the conclusions drawn in Chapter 5 pertain to the whole thesis.

[13] See citations: Montén and Thater (2010) and Montén and Thater (2011).

Chapter 2

Ageing municipalities, gerontocracy and fiscal competition

2.1 Introduction

Low birth rates and increased longevity lead to population ageing in many developed countries.[1] This demographic transition will have wide-reaching political, economic and social consequences. Economics literature has so far mostly discussed the macroeconomic impact of ageing societies. This chapter focuses on the local level and asks how demographic change will affect the provision of publicly provided goods. On the one hand, as the median voter's age increases the provision of publicly provided goods may shift toward those goods appreciated mostly by the elderly population. On the other hand, the competition among municipalities for the young and mobile intensifies, thus forcing the municipalities to provide more of those goods that make the location attractive for the younger population. In a model of fiscal competition among gerontocratic municipalities, we demonstrate that goods

[1] This chapter is based on the article "Ageing municipalities, gerontocracy and fiscal competition", as well as on the working paper with the same title, both co-authored by Prof. Marcel Thum and myself (see Montén and Thum (2008) and Montén and Thum (2010)).

for the elderly may be excessively provided in the early phase of a gerontocratic regime. However, when the share of the old population becomes sufficiently large, the effect of fiscal competition dominates and an inefficiently high amount of publicly provided goods for the young population is supplied.

In the next decades, many European economies will undergo significant demographic changes. According to UN projections (United Nations, 2007), the countries in central and eastern Europe will experience the most rapid population loss. In southern and western Europe, the more significant demographic process will be ageing. Germany's population will only shrink by 0.12 percent by 2020.[2] However, the median age is projected to increase by as much as 5.2 years from 42.1 years in 2005 to 47.3 years in 2020. Then only Italy is projected to have an even older population with a median age of 47.5 years. These shifts in the size and age structure can have significant consequences for the economy as a whole. In particular, the impact on capital markets (e.g., Abel, 2001; Börsch-Supan *et al.*, 2002; Krueger and Ludwig, 2007; Poterba, 2001, 2004) and on public pension schemes (e.g., Breyer and Stolte, 2001; Casamatta *et al.*, 2001; Demange and Laroque, 1999; Fehr, 2000; Sinn and Uebelmesser, 2002) has been extensively studied.[3] The focus on the aggregate demography often hides that even more pronounced demographic changes occur on the regional or local level.

The changing size and composition of the population poses many challenges for local policy makers. Downsizing of the infrastructure is needed in order to maintain fiscal balances. The portfolio of publicly provided goods has to be adjusted while taking into account the changing age structure. While the younger population may mostly desire access to jobs, schools or

[2] All values pertain to the medium variant of the 2006 revision of the World Population Prospects Database by the Population Division of the Department of Economic and Social Affairs of the United Nations Secretariat, accessible at: http://esa.un.org/unpp.

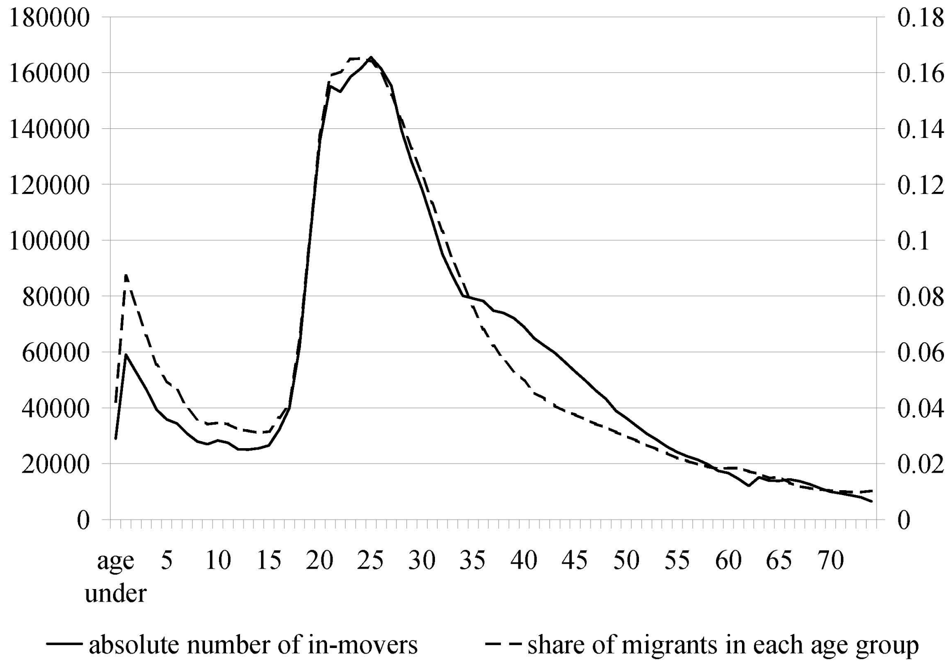
[3] Breyer and Stolte (2001) and Sinn and Uebelmesser (2002) take a political economy point of view on public pension reforms. Similar this chapter, Sinn and Uebelmesser (2002) consider the power of the old in a gerontocratic society. According to their calculations, Germany will be a gerontocracy in 2016 as pension reforms benefiting the younger cohorts will then become politically unfeasible.

child care facilities, the elderly may prefer affordable public transport and access to specific recreational opportunities and cultural offerings. The necessary adjustments in the public budget may create conflicts between generations. As the median age increases, majority voting outcomes may shift to benefit the elderly population at the cost of the younger generations. Such generational conflicts in local service provision have been addressed in other contributions, in particular with respect to the provision of public education (e.g. Poterba (1998) for the US and Grob and Wolter (2005) and Cattaneo and Wolter (2009) for Switzerland). Konrad (1995) shows that gerontocracies have an excessive incentive to invest in immobile infrastructure as opposed to mobile human capital. Our model also contains a strategic investment motive on a subnational level. Haupt and Peters (2003) consider an overlapping generations model in which the contribution rates to public pension schemes lead to interregional competition. When the young can strategically migrate, their exploitation by the gerontocracy is limited. The exit option of migration also plays a crucial role in our model. Finally, the studies by Borge and Rattsø (1995); Borge and Rattsø (2008) are closely related to this chapter. They mainly focus on local services, such as child care and elderly care, and analyze the negative effect of ageing on the per capita spending on services for the younger cohorts. In contrast to this chapter, the influence of local interaction and fiscal competition plays no role.

Fiscal competition in the context of generational conflicts arise, since the young population is mobile, while the old are immobile. This is corroborated by empirical evidence on intermunicipal mobility. Figure 2.1 illustrates the age distribution of intermunicipal migration in Germany in 2007. The solid line depicts the absolute number of in-migrants (left scale). The dashed line shows the percentage of intermunicipal migrants in each age group (migrants at age t / population at age t, right scale). Most migration occurs between age 20 and 35 when young people relocate because of job opportunities and family formation. Given the decision to move, the ensuing choice is made between alternative municipalities, e.g., in the vicinity of the workplace. Here, local amenities and publicly provided services clearly play a decisive role.

Our contribution examines the effects ageing has on the provision of pub-

Figure 2.1: Age distribution of intermunicipal migrants in Germany (2007)



Source: Federal Statistical Office

licly provided goods at the local level. The young population faces a threat once the elderly gain the majority, as a gerontocracy wants to provide less of the impure public goods for the young. However, the young will finally receive even more than under welfare maximization, if the share of elderly is sufficiently large. This seemingly counterintuitive result appears as fiscal competition for the young intensifies. As municipalities are often financed to a large extent through per capita grants, the local budget depends on the population size. To attract young families, even gerontocratic municipalities are forced to provide goods for younger cohorts. When ageing proceeds, fiscal competition becomes so intense that the level of the publicly provided goods for the young even exceeds the welfare maximizing level.

We also show that the initial distribution of the population effects the outcome. The paradoxical case where the gerontocracies provide too much of the publicly provided good for the young is more likely when the elderly population is more unequally distributed across municipalities. Regions with large disparities in their age structures experience a more intense fiscal competition for the young.

Section 2 describes the setup of the model. In Section 3, we discuss the equilibrium outcomes for gerontocracies that face fiscal competition. Section 4 derives the social planner's solution. In Section 5, the outcomes from the previous sections are contrasted with one another and changes in the distribution of the elderly are considered. Section 6 discusses alternative variants and extensions of our approach. Section 7 concludes.

2.2 The model

We consider a region with two municipalities. The population of this region consists of both young and old individuals. The fundamental difference between the two population groups is that the young are able to choose the municipality in which they reside, whereas the old segment of the population is immobile. Each municipality supplies two congestible public goods, one for each population segment. The funding for the publicly provided goods is achieved exclusively through per capita grants. When the population increases, a municipality receives more resources for financing its services. However, when there are more users the individual benefit from the publicly provided good is reduced due to crowding effects. Since the young are mobile between municipalities, their utility maximizing behavior places the municipalities in competition with one another.

2.2.1 Municipal budgets

There is a fixed total young population, N, and each young person must live in one of the two municipalities. Therefore, each municipality has a young population N_i where $i = 1, 2$ stand for the respective municipality and accordingly $N = N_1 + N_2$. Additionally each municipality also has a number of elderly individuals M_i where $i = 1, 2$.

The size of the publicly provided goods for the young and the old are denoted by Y_i and X_i, respectively. The prices of the two goods are normalized to unity. These publicly provided goods have to be financed in each municipality by a transfer from an upper level of government. The municipality receives a per capita block grant b.[4] The municipal budget constraint thus has the following form:

$$b(N_i + M_i) = Y_i + X_i, \quad \text{where} \quad i = 1, 2. \tag{2.1}$$

The assumption of per capita grants captures a stylized fact of many municipal financing schemes in continental Europe. Even when there is some degree of tax autonomy, revenue sharing and equalization schemes make the local budgets de facto dependent on the population size (OECD, 2005). Transfers and grants are intended to achieve horizontal equity in per capita revenue even when tax base disparities exist. We abstract from the financing of the grants and only consider the allocation of the funds. The introduction of tax autonomy and local financing through distortive taxation could clearly change some details of the analysis.

2.2.2 Age group specific preferences

In each municipality, the utility of a young individual depends on the amount of publicly provided goods (Y_i) and on the size of the population in this age group N_i. In particular, we assume the following utility function:

$$U_i = \left(\frac{Y_i}{N_i^\alpha}\right)^\beta \quad i = 1, 2, \tag{2.2}$$

where $\beta \in (0, 1)$ ensures declining marginal utility. The parameter $\alpha \in [0, 1]$ determines the degree of congestion. In the extreme cases when $\alpha = 0$ and $\alpha = 1$, the publicly provided good has the characteristic of a pure public good or a private good, respectively. The intermediate cases, $0 < \alpha < 1$, capture the empirically relevant phenomenon of partial crowding for many publicly provided goods (Bergstrom and Goodman, 1973; Borcherding and Deacon, 1972).[5] The characteristic of the publicly provided goods are assumed to be

[4] The results of the model do not depend on the simplifying assumption of a single source of finance. The per capita grant only has to be the marginal source of finance.

[5] Edwards (1990) discusses the virtues of alternative crowding specifications and finds

the same in two adjacent communities. For simplicity, we exclude spillover effects between the two municipalities, i.e. the utility from locating in municipality i is independent of the level of provision in municipality j. This may seem to be an extreme assumption at first sight. However, most spillovers emerge from city centers to the benefit of suburban municipalities. Spillovers between suburban municipalities are probably negligible. As we consider competition between two similar municipalities in suburban or rural areas the exclusion of spillovers is reasonable. We also do not consider commuting for consumption, but focus on locally provided services available to the residents of a municipality.

The utility function of a representative old person is analogous to that of a young individual:

$$V_i = \left(\frac{X_i}{M_i^{\alpha}}\right)^{\beta} \quad i = 1, 2. \tag{2.3}$$

For simplicity, we assume identical parameters α and β for the young and the old.[6]

2.2.3 Location choice

Fiscal competition takes place over the bundle of publicly provided goods offered in each of the two municipalities. The municipalities set their levels of publicly provided goods knowing that young individuals can move to the municipality offering the better bundle for the young. For simplicity, we assume costless mobility. Since the relocation decision is intraregional in our case, the costs (both monetary and emotional) associated with mobility

that this simple decreasing marginal returns specification fares fairly well. Refer to Reiter and Weichenrieder (1997) for a survey of demand estimates of local public goods as well as a discussion of the measurement of crowding. The empirical studies find that the degree of crowding is close to unity for many locally provided public goods.

6 By construction, the model does not only represent the potential conflict of interest arising between young and old. Instead the framework may also be interpreted in terms of any two groups that demand different publicly provided goods and differ in terms of mobility (e.g., families with children and single households). See Section 6 for a brief discussion of intergenerational conflicts arising from intertemporal considerations.

should be relatively low. As more young move in, the congestion of the public good will reduce the benefits from the publicly provided good.[7] The migration equilibrium is achieved, when a young person is just indifferent between the two municipalities:

$$U_1(N_1, Y_1) = U_2(N_2, Y_2). \tag{2.4}$$

It can easily be verified that the migration equilibrium is stable. Using $N_2 = N - N_1$, the young population in municipality i amounts to

$$N_i^E = \frac{N}{\left(\frac{Y_j}{Y_i}\right)^{\frac{1}{\alpha}} + 1} \tag{2.5}$$

with $i \neq j$. The equilibrium population in municipality i is only dependent on the provision levels for the young in each municipality, as well as on the total population of young.

2.3 Gerontocracies

When the elderly are in the majority and have full authority over the allocation decision, a municipality will be considered a gerontocracy. The elderly choose the utility maximizing level of the publicly provided good. Full exploitation of the young generation is prevented by the mobility of the young. Each gerontocracy tries to provide a sufficient level of publicly provided goods for the young to remain competitive and to generate a sufficiently large budget. Hence, the elderly maximize the budget that can be allocated to their own publicly provided good:

$$\max_{Y_i} \quad X_i = b \cdot (M_i + N_i^E(Y_i)) - Y_i. \tag{2.6}$$

The municipality receives b per inhabitant. The size of the young population

[7] We focus on the provision of publicly provided goods as the sole determinants of location choices. In addition (or alternatively), production may be added to the model. Decreasing marginal productivities limit the inflow of workers and also generate an interior migration equilibrium.

depends on the amount of the publicly provided good Y_i [see Eq. (2.5)]. To obtain the budget available for the publicly provided good of the elderly, the expenditure for the young Y_i is subtracted from the total budget. From the perspective of a single gerontocracy, the optimal provision of the publicly provided good for the young is implicitly given by

$$\frac{\partial X_i}{\partial Y_i} = b \cdot \frac{N}{\left[\left(\frac{Y_j}{Y_i}\right)^{\frac{1}{\alpha}} + 1\right]^2} \cdot \frac{Y_j^{\frac{1}{\alpha}}}{\alpha \cdot Y_i^{\frac{1}{\alpha}+1}} - 1 = 0. \tag{2.7}$$

For $\alpha > \frac{1}{2}$, there is a unique pure strategy equilibrium[8] with

$$Y_1^G = Y_2^G = \frac{bN}{4\alpha}. \tag{2.8}$$

The provision for the young in either municipality does not depend on the share of elderly, but only on the total number of young across both municipalities. Fiscal competition forces the gerontocratic municipalities to spend more on the young population when the per capita transfer b is high, when the young population N is large and when the crowding effects are low (low α). Using the budget constraint, the equilibrium level of the publicly provided good for the elderly X_i, $i = 1, 2$ amounts to:

$$X_i^G = b\left(M_i + \frac{N}{2}\left(1 - \frac{1}{2\alpha}\right)\right). \tag{2.9}$$

As $\frac{1}{2} < \alpha < 1$, the public provision of the good for the elderly does not only increase in the size of the old population M_i but also in the size of young population N as the gerontocracy diverts some resources from the young.

To analyze the impact of ageing on the provision of publicly provided goods, we normalize the size of the total population to unity: $N + M = 1$. The share of the elderly in the total population is then defined as $s \equiv \frac{M}{1}$. For notational convenience, we furthermore introduce m_i as the share of the elderly population living in municipality i: $m_i \equiv \frac{M_i}{M_i + M_j}$. These definitions allow us to write the equilibrium utilities of the young and old in municipality

[8] For $\alpha \leq 1/2$, no pure strategy equilibrium exists. The details of the derivation and the properties of the mixed strategy equilibrium can be found in 2.A.

i as

$$U_i^G = \left(\frac{b \cdot (1-s)^{1-\alpha}}{2^{2-\alpha} \cdot \alpha} \right)^{\beta} \tag{2.10}$$

and

$$V_i^G = \left\{ b \cdot \left[(m_i \cdot s)^{1-\alpha} + (m_i \cdot s)^{-\alpha} \cdot \frac{1-s}{2} \cdot \left(1 - \frac{1}{2\alpha} \right) \right] \right\}^{\beta}, \tag{2.11}$$

respectively.

Taking the derivatives of equations (2.10) and (2.11) with respect to s yields information on whether the young and the old gain or lose from an ageing society. The utility of the young clearly declines when the share of the elderly increases. The impact on the utility of the elderly, however, is less obvious:

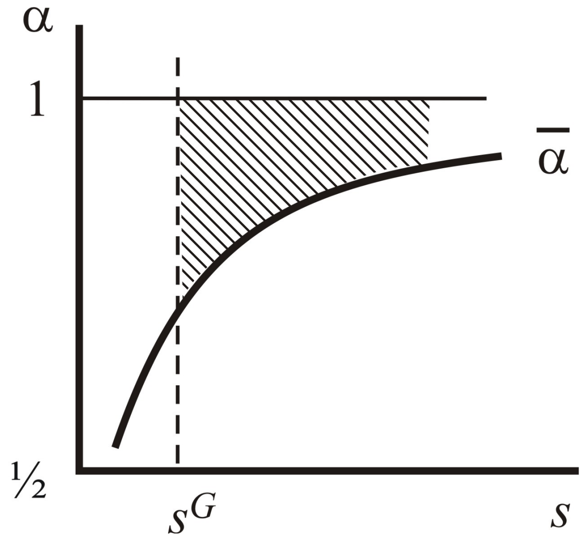
$$\begin{aligned} \frac{\partial V_i^G}{\partial s} =& \beta \cdot \left[V_i^G \right]^{\beta-1} \cdot b \cdot m_i^{-\alpha} \cdot s^{-\alpha-1} \cdot \\ & \left\{ s \cdot (1-\alpha) \cdot \left[m_i - \frac{1}{2} \left(1 - \frac{1}{2\alpha} \right) \right] - \frac{\alpha}{2} \left(1 - \frac{1}{2\alpha} \right) \right\}. \end{aligned} \tag{2.12}$$

The impact of ageing on the utility of the elderly depends on the expression inside the curly brackets.

Proposition 2.1. *In gerontocratic municipalities that face fiscal competition ageing decreases (increases) the utility of the elderly population if the degree of crowding exceeds (remains below) a critical level $\bar{\alpha}$.*

Proof. The impact of ageing on the utility of the elderly depends on the sign of $z \equiv s \cdot (1-\alpha) \cdot \left[m_i - \frac{1}{2} \left(1 - \frac{1}{2\alpha} \right) \right] - \frac{\alpha}{2} \left(1 - \frac{1}{2\alpha} \right)$. For $\alpha = \frac{1}{2}$, we get $z(\frac{1}{2}) = \frac{sm_i}{2} > 0$. $\alpha = 1$ yields $z(1) = -\frac{1}{4}$. As z is strictly declining in α ($\frac{\partial z}{\partial \alpha} = -sm_i - \frac{s}{4\alpha^2} - \frac{1-s}{2} < 0$), there is exactly one critical value $\bar{\alpha}$ for which $\frac{\partial V_i^G}{\partial s} \gtreqless 0 \Leftrightarrow \alpha \lesseqgtr \bar{\alpha}$.

Figure 2.2: The impact of ageing on the elderly in a gerontocracy



The outcome is illustrated in Figure 2.2. We measure the share of the elderly s on the horizontal axis and the degree of crowding α on the vertical axis. s^G denotes the minimum share of the elderly in society as a whole, necessary to make municipality i a gerontocracy ($s \cdot m_i \geq \frac{1}{2}(1-s) \Rightarrow s \geq s^G = \frac{1}{2m_i+1}$). In a gerontocratic municipality, the gains or losses of the elderly depend on both their share (s) and the characteristics of the publicly provided good (α). The elderly lose from an ageing society for all parameter values α above $\bar{\alpha}$; when $\alpha < \bar{\alpha}$, the old gain from ageing. The elderly are more likely to lose from an ageing society when the publicly provided good exhibits strong crowding effects (high α) because ageing implies that the transfers from young to old have to be shared among a larger number of elderly people. Put differently: If the publicly provided good is closer to a public good, the benefits of the good can be shared among more elderly. As the size of the publicly provided good for the elderly grows with an ageing society, the utility of each old person will increase. Note also that the elderly population in smaller municipalities is more likely to lose from the ageing process; a reduction in the relative size of the elderly in municipality i shifts the $\bar{\alpha}$-curve downwards and increases the shaded area ($\partial\bar{\alpha}/\partial m_i > 0$).

In the following, we will derive the social planner's solution and compare the outcome with the resource allocation and utilities derived by the two population groups in a gerontocratic society.

2.4 Welfare maximization

We consider the decision of a utilitarian social planner who maximizes welfare by allocating the budget across municipalities and across the two types of publicly provided goods. The social planner is not restricted to the budget constraints of the individual municipalities but has to obey the aggregate budget constraint. In welfare maximization we consider two alternative distributions of the young population. First, we consider the first-best solution where provision for the young only takes place in one municipality. Second, we consider the case with the additional constraint of "equal living conditions", where the young population is equally divided between the two municipalities. Qualitatively the results do not differ. The two different welfare maximization problems are solved in turn, first the case of young concentrated in one municipality, and then under equal division of the young population. Each section is immediately followed by a comparison to the gerontocracy.

2.4.1 Young concentrated in one municipality

In this case, the mobile young choose the municipality that grants the highest utility and the social planner has no power over individual migration decisions. Alternatively, we assume in Section 2.4.3 that the social planner maximizes welfare for a given distribution of the young population, i.e. half of the young population resides in each municipality as is the case in the symmetric Nash equilibrium. We discuss additional alternative specifications of the social planner in Section 2.5.

The social planner maximizes the utilitarian welfare function

$$W = N_1 U_1 + N_2 U_2 + M_1 V_1 + M_2 V_2$$

by choosing the sizes of the publicly provided goods for the young Y_i and for the elderly X_i ($i = 1, 2$). She has to take into account the budget constraint[9]

$$Y_1 + Y_2 + X_1 + X_2 = b \cdot (N_1 + N_2 + M_1 + M_2).$$

Since there are economies of scale in the provision of the public services ($\alpha < 1$) and the young are perfectly mobile, it is always optimal for the social planner to provide the public services for the young in one municipality only. The young will be concentrated in this municipality. Hence, the social planner's maximization problem boils down to

$$\max_{Y, X_1, X_2} \quad W = NU + M_1 V_1 + M_2 V_2$$

$$\text{s.t.} \quad Y + X_1 + X_2 = b \cdot (N + M_1 + M_2).$$

The subscript on Y is dropped because the young reside in one community only.

Maximization of the welfare function yields the following provision levels:

$$Y^W = \frac{b}{N^\gamma + M_1^\gamma + M_2^\gamma} \cdot N^\gamma \tag{2.13}$$

$$X_i^W = \frac{b}{N^\gamma + M_1^\gamma + M_2^\gamma} \cdot M_i^\gamma \tag{2.14}$$

with $i = 1, 2$ and $\gamma \equiv \frac{1-\alpha\beta}{1-\beta}$. Note that, due to $\alpha > 1/2$, we always have $\gamma > 1$. The provision levels depend on the aggregate budget constraint and on the relative share of the population groups. The provision level increases in the size of a population group. Hence, with ageing, the social planner will provide more of the publicly provided goods for the elderly and less for the young.

The more interesting question is how the utilities of the different groups are affected by an ageing society. Using the notation for the share of the elderly s and the distribution of the elderly m_1, m_2, we obtain the utility

[9] Even though we have normalized the total population to unity, we explicitly keep the size of the distinct population groups in the formula to facilitate interpretation.

levels as

$$U^W = \left[\frac{b \cdot (1-s)^{\frac{1-\alpha}{1-\beta}}}{(1-s)^\gamma + s^\gamma \cdot (m_1^\gamma + m_2^\gamma)}\right]^\beta \tag{2.15}$$

$$V_i^W = \left[\frac{b \cdot (s \cdot m_i)^{\frac{1-\alpha}{1-\beta}}}{(1-s)^\gamma + s^\gamma \cdot (m_1^\gamma + m_2^\gamma)}\right]^\beta \tag{2.16}$$

for $i = 1, 2$. As we have seen before, a decreasing young population will lead to lower provision levels. However, the good also has to be shared among fewer young inhabitants. For the elderly, the larger group size leads to higher provision levels. The larger publicly provided good has to be shared among more users. The net effect of ageing on the utility of the young and the old is summarized in

Proposition 2.2. *(a) With welfare maximization, ageing leads to a lower provision of the publicly provided good for the young and a higher provision for the elderly. (b) Ageing causes a decline of the utility of the young population, whereas the effect on the utility of the old population is ambiguous.*

Proof. (a) Immediately follows from the derivatives of (2.13) and (2.14). (b) Taking the derivative of U^W with respect to s leads to

$$\frac{\partial U^W}{\partial s} \gtreqless 0 \Leftrightarrow \alpha \cdot (1-s)^\gamma + s^\gamma (m_1^\gamma + m_2^\gamma)(\alpha - \frac{\gamma}{s}) \gtreqless 0. \tag{2.17}$$

We evaluate the left-hand side at the maximum and show that it will never exceed 0. Note that $\alpha - \gamma/s < 0$. Hence, the left-hand side will be maximized for $m_1 = m_2 = \frac{1}{2}$. As the expression grows in α, it has to be evaluated at $\alpha = 1$, which implies $\gamma = 0$. The left-hand side becomes zero at the maximum and therefore $\frac{\partial U^W}{\partial s} \leq 0$. For the elderly, the sign of the derivative of V_i will depend on

$$\frac{\partial V_i^W}{\partial s} \gtreqless 0 \Leftrightarrow (1-s)^{\gamma-1}\left[\frac{1-\alpha}{1-\beta} + \alpha s\right] - \alpha s^\gamma (m_1^\gamma + m_2^\gamma) \gtreqless 0. \tag{2.18}$$

For very old societies ($s \to 1$), the derivative becomes negative. Hence, the elderly lose from further ageing. For $s = \frac{1}{2}$ and an equally distributed elderly population $m_1 = m_2 = \frac{1}{2}$, the derivative is strictly positive. Here, the elderly gain from an ageing society.

As in the case of a gerontocracy, the elderly gain for a certain combination of ageing (s) and crowding (α). However, beyond a certain threshold they lose. In Section 5, we compare the impact of ageing on the resource allocation under the social planner and the gerontocrat.

The above approach also allows us to analyze the consequences of heterogeneities in the ageing process. One of the main challenges for ageing societies is the large heterogeneity among municipalities. Whereas one region is still fairly young, the other already has a large number of elderly. How will differences in the number of elderly affect the provision of publicly provided goods? Without loss of generality, let municipality 1 have more old inhabitants than municipality 2: $m_1 = 0.5 + \eta$ and $m_2 = 0.5 - \eta$ with $\eta \in (0, 0.5]$.

Proposition 2.3. *A more unequal distribution of the elderly population reduces the provision of the publicly provided good for the young and, therefore, the utility of this group.*

Proof. We rewrite (2.13) as

$$Y^W = \frac{b \cdot (1-s)^\gamma}{(1-s)^\gamma + s^\gamma[(0.5+\eta)^\gamma + (0.5-\eta)^\gamma]}. \tag{2.19}$$

Differentiating with respect to η yields

$$\frac{\partial Y^W}{\partial \eta} = -Y^W \frac{s^\gamma \gamma[(0.5+\eta)^{\gamma-1} - (0.5-\eta)^{\gamma-1}]}{(1-s)^\gamma + s^\gamma[(0.5+\eta)^\gamma + (0.5-\eta)^\gamma]} < 0. \tag{2.20}$$

As the size of the young population is fixed, the utility of this group $\left(\frac{Y^W}{(1-s)^\alpha}\right)^\beta$ must also decline when heterogeneity increases.

This result suggests that the young population will suffer more from an ageing society if the elderly population is more unequally distributed across municipalities. An unequal distribution of the immobile old makes it more expensive for the social planner to provide the publicly provided good for the elderly. As a consequence, the provision of the publicly provided good for the young is reduced. The impact of a more unequal population distribution on the elderly is ambiguous. Depending on the size of the elderly population (s) and on the degree of heterogeneity (η), the elderly population may gain or lose when the distribution becomes more unequal. The rising cost of provision may be offset by the changing group size.

Due to economies of scale, the provision of the publicly provided good for the mobile young will only take place in one of the two municipalities under welfare maximization. However, this result may not be in line with regional policy which stipulates "equal living conditions" or "equal opportunities" in all regions of a country. By requiring provision for the young in both municipalities (e.g., assuming $Y_1 = Y_2$), the effect of such policy can easily be integrated into the current framework. The additional constraint forces the social planner to lower the provision for both young and old. The qualitative effects of ageing, however, remain the same. In Section 2.4.3 this additional constraint is imposed, and the results are discussed in detail. However, in the subsequent section the welfare solution where all young are concentrated in one municipality is compared to the gerontocracy outcome.

2.4.2 Comparison of the gerontocracy and welfare maximization, young concentrated in one

The two preceding sections have shown that, in gerontocracies as well as with welfare maximization, ageing leads to a lower provision of publicly provided goods for the young and a higher provision for the elderly. The dominance of the elderly *per se* would suggest that the public good for the young is provided on a minimum scale only. However, the exploitation of the young is limited as the municipalities have to compete for the mobile young. This raises the question whether the provision of the publicly provided good for the young remains on an inefficiently low level in gerontocracies or whether, due to fiscal competition, the provision may even exceed the welfare maximizing level.

In order to compare the provision levels in the two scenarios (gerontocracy and social planner with the young population concentrated in one municipality), we rewrite the provision level for the young in a gerontocracy [see Eq. (2.8)] as

$$Y^G = \frac{b \cdot (1-s)}{4\alpha} \tag{2.21}$$

and in the case of a social planner [see Eq. (2.13)] as

$$Y^W = \frac{b \cdot (1-s)^\gamma}{(1-s)^\gamma + s^\gamma \cdot (m_1^\gamma + m_2^\gamma)}. \tag{2.22}$$

Note that the comparison is carried out for gerontocratic societies ($s \geq \frac{1}{2}$) and for publicly provided goods with some crowding ($\alpha \in [0.5, 1]$). Rearranging the terms leads to

$$Y^W \gtreqless Y^G \Leftrightarrow [4\alpha - (1-s)] \cdot (1-s)^{\gamma-1} \gtreqless s^\gamma \cdot (m_1^\gamma + m_2^\gamma) \tag{2.23}$$

This comparison yields:

Proposition 2.4. *The publicly provided good for the young is provided on an inefficiently low level for 'young' gerontocracies but is provided on an inefficiently large scale for 'old' gerontocracies.*

Proof. Define $lhs \equiv [4\alpha - (1-s)] \cdot (1-s)^{\gamma-1}$ and $rhs \equiv s^\gamma \cdot (m_1^\gamma + m_2^\gamma)$. The right-hand side of (2.23) increases in s ($\frac{\partial rhs}{\partial s} > 0$). Differentiating the left-hand side of (2.23) yields $\frac{\partial lhs}{\partial s} = (1-s)^{\gamma-2} \cdot [-(\gamma-1)4\alpha + \gamma(1-s)]$. The left-hand side of (2.23) has a maximum at $s = 1 - \frac{\gamma-1}{\gamma}4\alpha < 1$, as $\frac{\partial^2 lhs}{\partial s^2} < 0$. We have $lhs > rhs$ at $s = 0.5$ and $lhs < rhs$ at $s = 1$. Hence, there is one critical level s_0 for which $s \gtreqless s_0 \Leftrightarrow Y^G \gtreqless Y^W$.

When the elderly gain the majority in a society, a gerontocracy will initially use its power to exploit the young. The provision of the publicly provided good for the young remains on an inefficiently low level. Accordingly, the provision of the publicly provided good for the elderly is excessively large. However, when the society grows older, fiscal competition eventually forces the gerontocracy to provide more of the publicly provided good than the social planner.

This results in the critical level s_0, beyond which an ageing society provides an excessive amount of publicly provided goods for the young. The variable s_0 can be used to carry out some interesting comparative statics. Let

$$e \equiv [4\alpha - (1 - s_0)] \cdot (1 - s_0)^{\gamma-1} - s_0^{\gamma} \cdot ((0.5 + \eta)^{\gamma} + (0.5 - \eta)^{\gamma}) \tag{2.24}$$

where we have replaced m_1 and m_2 with $0.5 + \eta$ and $0.5 - \eta$, respectively, to analyze the impact of a more unequal distribution of the elderly population. Differentiating (2.24) immediately yields $\frac{\partial e}{\partial s_0} < 0$ (see the Proof to Proposition 2.4) and $\frac{\partial e}{\partial \eta} < 0$. Differentiating with respect to the crowding parameter α leads to

$$\begin{aligned}\frac{\partial e}{\partial \alpha} =& 4(1 - s_0)^{\gamma-1} - \frac{b}{1-b} s^{\gamma} \cdot \\ & \{(0.5 + \eta)^{\gamma} [ln(1 - s_0) - ln(s_0) - ln(0.5 + \eta)] + \\ & (0.5 - \eta)^{\gamma} [ln(1 - s_0) - ln(s_0) - ln(0.5 - \eta)]\}.\end{aligned} \tag{2.25}$$

Numeric evaluation of this expression shows that $\frac{\partial e}{\partial \alpha} > 0$. Hence, we get the following comparative statics results:

$$\frac{ds_0}{d\eta} = -\frac{\partial e/\partial \eta}{\partial e/\partial s_0} < 0 \tag{2.26}$$

and

$$\frac{ds_0}{d\alpha} = -\frac{\partial e/\partial \alpha}{\partial e/\partial s_0} > 0. \tag{2.27}$$

The excessive provision of the publicly provided good for the young is reached earlier, i.e. with a lower share of the elderly s, when the population is more unequally distributed across the municipalities (η) and when the publicly provided good exhibits less crowding (lower α).

So far we have focused on a comparison of the provision levels. However, since the provision level is dependent on the number of users, the utility levels achieved by the population should be considered. We can immediately state that the utility of the young will be higher in a gerontocracy if the gerontocracy provides more publicly provided goods for the young than the social planner. Note that the gerontocracies provide the publicly provided good in each municipality whereas the social planner concentrates her expenditures for the young in one municipality. Hence, the larger publicly provided good has to be shared among fewer young people in a gerontocracy. In this case, the young must be better off in a gerontocracy. Even with a slightly smaller provision level in gerontocracies, the young may gain due to the fiscal competition effect. To obtain the general condition for the young being better off in a gerontocracy, we compare the utility levels in the two scenarios. The utility of the young is

$$U_i^G = \left(\frac{b \cdot (1-s)^{1-\alpha}}{2^{2-\alpha} \cdot \alpha} \right)^{\beta} \tag{2.28}$$

in a gerontocracy [see Eq. (2.10)] and

$$U^W = \left(\frac{b \cdot (1-s)^{\frac{1-\alpha}{1-\beta}}}{(1-s)^{\gamma} + s^{\gamma} \cdot (m_1^{\gamma} + m_2^{\gamma})} \right)^{\beta} \tag{2.29}$$

with a social planner [see Eq. (2.15)]. The comparison of the utility of a young person in a gerontocracy and in a welfare maximizing society then yields

$$U^W \gtreqless U^G \Leftrightarrow \left[2^{2-\alpha}\alpha - (1-s) \right] \cdot (1-s)^{\gamma-1} \gtreqless s^{\gamma} \cdot (m_1^{\gamma} + m_2^{\gamma}) \,. \tag{2.30}$$

This comparison has the same qualitative structure as Eq. (2.23), where the provision quantities of the two scenarios are compared. Let $\bar{s}$ denote the critical share of the elderly where the utilities the young achieve are the same in a gerontocracy and in a welfare maximizing society ($U^W = U^G$). As we have $2^{2-\alpha} \leq 4$, we immediately get $\bar{s} \leq s_0$. This implies that $U^G > U^W$ occurs 'sooner', i.e. at a lower s, than for the case when $Y^G > Y^W$. The young arrive at inefficiently low utility levels in moderate gerontocracies. However, when society grows older, the young even benefit. Due to fiscal competition, their utility exceeds the welfare maximizing level.

2.4.3 Equal division of the young population between the two municipalities

In contrast to the previous section, where public provision for the young only took place in one community, here the a constraint to ensure provision in both communities is added. To obtain a fair comparison with the gerontocratic scenario, we assume that the social planner has to obey a rule of equal living conditions, i.e., she has to provide the same amount of the publicly provided good for the young in both municipalities ($Y = Y_1 = Y_2$). Such constraints are frequently imposed to ensure equal opportunities for the young across regions. As was shown in the previous Section 2.4.1, without such a constraint, the social planner would provide the public good for the young in only one municipality to benefit from economies of scale. To isolate the distortions emerging from fiscal competition of gerontocracies, we force the social planner to obey this constraint of equal living conditions. An alternative justification of a uniform Y is to have the same population distribution in the social planner's problem as in the gerontocratic setting. The mobile young choose the municipality that grants the highest utility. As the size of the public good is the same, the young will be equally distributed across the two municipalities in equilibrium.

The social planner maximizes the utilitarian welfare function

$$W = N_1 U_1 + N_2 U_2 + M_1 V_1 + M_2 V_2$$

by choosing the sizes of the publicly provided goods for the young Y_i and for the elderly X_i ($i = 1, 2$). She has to take into account the budget constraint

$$Y_1 + Y_2 + X_1 + X_2 = b \cdot (N_1 + N_2 + M_1 + M_2).$$

Due to the constraint of equal living conditions for the young, the social planner's maximization problem boils down to

$$\max_{Y,X_1,X_2} \quad W = N\left(\frac{Y}{(N/2)^{\alpha}}\right)^{\beta} + M_1\left(\frac{X_1}{M_1^{\alpha}}\right)^{\beta} + M_2\left(\frac{X_2}{M_2^{\alpha}}\right)^{\beta}$$

$$\text{s.t.} \quad 2Y + X_1 + X_2 = b \cdot (N + M_1 + M_2).$$

The subscript on Y is dropped because the provision level is identical across the two municipalities. Maximization of the welfare function yields the following provision levels:

$$Y^W = \frac{b(N + M_1 + M_2)}{2\left(\frac{N}{2}\right)^{\gamma} + M_1^{\gamma} + M_2^{\gamma}} \cdot \left(\frac{N}{2}\right)^{\gamma} \tag{2.31}$$

$$X_i^W = \frac{b(N + M_1 + M_2)}{2\left(\frac{N}{2}\right)^{\gamma} + M_1^{\gamma} + M_2^{\gamma}} \cdot M_i^{\gamma} \tag{2.32}$$

with $i = 1, 2$ and $\gamma \equiv \frac{1-\alpha\beta}{1-\beta}$. Note that due to $\alpha < 1$, we always have $\gamma > 1$. The provision levels depend on the aggregate budget constraint, on the relative size of the population groups and on the distribution of the elderly across municipalities.

To isolate the impact of ageing on the provision levels of the publicly provided goods and on the utilities of both generations we again normalize total population to unity. Using the notation s for the share of the elderly and m_1, m_2 for the distribution of the elderly, the provision quantities can be written as:

$$Y^W = \frac{b}{2\left(\frac{1-s}{2}\right)^{\gamma} + s^{\gamma}(m_1^{\gamma} + m_2^{\gamma})} \cdot \left(\frac{1-s}{2}\right)^{\gamma} \tag{2.33}$$

$$X_i^W = \frac{b}{2\left(\frac{1-s}{2}\right)^{\gamma} + s^{\gamma}(m_1^{\gamma} + m_2^{\gamma})} \cdot (sm_i)^{\gamma}. \tag{2.34}$$

The corresponding utility levels amount to

$$U^W = \left[\frac{b \cdot \left(\frac{1-s}{2}\right)^{\frac{1-\alpha}{1-\beta}}}{2\left(\frac{1-s}{2}\right)^{\gamma} + s^{\gamma} \cdot (m_1^{\gamma} + m_2^{\gamma})} \right]^{\beta} \tag{2.35}$$

$$V_i^W = \left[\frac{b \cdot (s \cdot m_i)^{\frac{1-\alpha}{1-\beta}}}{2\left(\frac{1-s}{2}\right)^{\gamma} + s^{\gamma} \cdot (m_1^{\gamma} + m_2^{\gamma})} \right]^{\beta} \tag{2.36}$$

for $i = 1, 2$. The effect of ageing can now be found by considering changes in the share of the elderly s. The results are summarized in

Proposition 2.5. *(a) With welfare maximization, ageing leads to a lower provision of the publicly provided good for the young and a higher provision for the elderly. (b) Ageing causes a decline in the utility of the young population. With ageing the utility of the old population increases, however when the share of the elderly becomes sufficiently high, the utility of the elderly decreases.*

Proof. (a) Immediately follows from the derivatives of (2.33) and (2.34). (b) Taking the derivative of U^W with respect to s leads to

$$\frac{\partial U^W}{\partial s} \gtreqless 0 \Leftrightarrow \alpha \cdot \left(\frac{1-s}{2}\right)^{\gamma} + s^{\gamma}(m_1^{\gamma} + m_2^{\gamma})(\alpha - \frac{\gamma}{s}) \gtreqless 0. \tag{2.37}$$

We evaluate the left-hand side at the maximum and show that it will never exceed 0. Note that $\alpha - \gamma/s < 0$. Hence, the left-hand side will be maximized for $m_1 = m_2 = \frac{1}{2}$. As the expression grows in α, it has to be evaluated at $\alpha = 1$, which implies $\gamma = 1$. The left-hand side amounts to $-\frac{1-s}{2} < 0$ at the maximum and therefore $\frac{\partial U^W}{\partial s} < 0$. For the elderly, the derivative of V_i^W with respect to s is

$$\frac{\partial V_i^W}{\partial s} \gtreqless 0 \Leftrightarrow \frac{\partial V_i^W}{\partial s} = V_i^W \left(\frac{\beta}{s}\right) \left[\frac{\gamma\left(\frac{1-s}{2}\right)^{\gamma-1}}{2\left(\frac{1-s}{2}\right)^{\gamma} + s^{\gamma}(m_1^{\gamma} + m_2^{\gamma})} - \alpha \right] \gtreqless 0. \tag{2.38}$$

The impact of ageing on the utility of the elderly depends on the sign of the term in the square brackets. We first evaluate this expression at the two extreme values of s. When $s = \frac{1}{2}$ the term in the brackets is positive and the utility of the elderly increases as ageing advances. However, when $s = 1$, then $\frac{\partial V_i^W}{\partial s} < 0$. Since the term in brackets is strictly declining in s, there is a unique threshold of s beyond which the utility of the elderly will decline.

Part (a) of Proposition 2.5 is not surprising. In an ageing society, the social planner will provide more of the publicly provided goods for the elderly and less for the young. The more interesting question is how the utilities of the different groups are affected by an ageing society [part (b) of Proposition 2.5]. As we have seen before, a decreasing young population will lead to lower provision levels. However, the good also has to be shared among fewer young inhabitants. Proposition 2.5 shows that the first effect dominates. The social planner lowers the provision for the young, which leads to a reduction in the utility level. For the elderly, the larger group size leads to higher provision levels. The larger publicly provided good, however, also has to be shared among more users. Proposition 2.5 demonstrates that the utility of the elderly initially increases with ageing, but decreases as the share of the elderly becomes sufficiently large.

Propositions 2.1 and 2.5 reveal similar results. Both under the social planner and the gerontocratic regimes, the elderly first gain when society ages but cease to benefit beyond a certain level of ageing. Even with a gerontocratic government, which pursues their own interests the old may lose. The fiscal competition among municipalities partially destroys the benefits of being in power. As these results are based on *changes* in utility, they do not necessarily reveal anything about the absolute utility of the elderly in ageing societies, i.e. whether the elderly are better off with a social planner rather than with a gerontocrat. In Section 2.4.4, we compare the utility levels with social planners and the gerontocrats in ageing societies.

Since the provision for the young depends on the size of the elderly populations in both municipalities, the above approach also allows us to analyze the consequences of heterogeneities in the ageing process. Similarly to Proposition 2.3 the heterogeneous distribution of the elderly can also be analyzed in this case. Without loss of generality, let municipality 1 have more old inhabitants than municipality 2: $m_1 = 0.5 + \eta$ and $m_2 = 0.5 - \eta$ with $\eta \in (0, 0.5]$.

Proposition 2.6. *A more unequal distribution of the elderly population reduces the provision of the publicly provided good for the young and, therefore, the utility of this group.*

Proof. We rewrite (2.33) as

$$Y^W = \frac{b \cdot \left(\frac{1-s}{2}\right)^\gamma}{2\left(\frac{1-s}{2}\right)^\gamma + s^\gamma[(0.5+\eta)^\gamma + (0.5-\eta)^\gamma]}. \tag{2.39}$$

Differentiating with respect to η yields

$$\frac{\partial Y^W}{\partial \eta} = -Y^W \frac{s^\gamma \gamma[(0.5+\eta)^{\gamma-1} - (0.5-\eta)^{\gamma-1}]}{2\left(\frac{1-s}{2}\right)^\gamma + s^\gamma[(0.5+\eta)^\gamma + (0.5-\eta)^\gamma]} < 0. \tag{2.40}$$

As the size of the young population is fixed, the utility of this group $U^W = \left(\frac{Y^W}{\left(\frac{1-s}{2}\right)^\alpha}\right)^\beta$ must also decline when heterogeneity increases.

Again this result indicates that the young population will suffer more from an ageing society, if the elderly population is more unequally distributed across municipalities. A more unequal distribution of the elderly makes it more expensive for the social planner to provide the publicly provided good and thus reduces provision for the young. Again the effect on the elderly is ambiguous.

2.4.4 Comparison of the gerontocracy and welfare maximization, equal division

In order to compare the provision levels in the two scenarios (gerontocracy and social planner), we rewrite the provision level for the young in a gerontocracy [see Eq. (2.8)] as

$$Y^G = \frac{b \cdot (1-s)}{4\alpha} \tag{2.41}$$

and in the case of a social planner we use Eq. (2.33). Note that the comparison is carried out for gerontocratic societies ($s \geq \frac{1}{2}$) and for publicly provided goods with some crowding ($\alpha \in [0.5, 1]$). This comparison yields

Proposition 2.7. *If the share of the elderly exceeds a critical threshold s_0, the publicly provided good for the young is provided on an inefficiently large scale.*

Proof. Rearranging the terms leads to

$$Y^W \gtreqless Y^G \Leftrightarrow 2 \cdot \left(\frac{1-s}{2}\right)^{\gamma-1} \cdot \left[\alpha - \frac{1-s}{2}\right] \gtreqless s^\gamma \cdot (m_1^\gamma + m_2^\gamma) \,. \tag{2.42}$$

Define $lhs \equiv 2 \cdot \left(\frac{1-s}{2}\right)^{\gamma-1} \cdot \left[\alpha - \frac{1-s}{2}\right]$ and $rhs \equiv s^\gamma \cdot (m_1^\gamma + m_2^\gamma)$. The right-hand side of (2.42) increases in s ($\frac{\partial rhs}{\partial s} > 0$). Differentiating the left-hand side of (2.42) yields $\frac{\partial lhs}{\partial s} = \left(\frac{1-s}{2}\right)^{\gamma-2} \cdot \left[-\gamma \cdot (\alpha - \frac{1-s}{2}) + \alpha\right]$. The left-hand side of (2.42) has a maximum at $s = 1 - \frac{\gamma-1}{\gamma} \cdot 2 \cdot \alpha < 1$, as $\frac{\partial^2 lhs}{\partial s^2} < 0$. We have $lhs > rhs$ at $s = 0$ and $lhs < rhs$ at $s = 1$. Hence, there is one critical level $s_0 \in (0, 1)$ for which $s \gtreqless s_0 \Leftrightarrow Y^G \gtreqless Y^W$.

The intuition is analogous to that of Proposition 2.4. Initially the gerontocracy can exploit the young by providing inefficiently low levels of public goods. However, when the society grows older, fiscal competition eventually forces the gerontocracy to provide more of the publicly provided good than the social planner.

As the distribution of the population across municipalities is the same in both the social planner and the gerontocracy scenarios, Proposition 2.7 also has immediate implications for the utility levels achieved by the young:

$$U^G \gtreqless U^W \Leftrightarrow s \gtreqless max(s_0, \frac{1}{2}). \tag{2.43}$$

If the publicly provided good is provided on a larger scale in gerontocracies, then the young gain from being ruled by the elderly and from the competition for young inhabitants.

Analogously to the comparison in Section 2.4.1 the comparative statics are carried out using the critical threshold s_0.[10] Let

$$e \equiv 2\left(\alpha - \frac{1-s_0}{2}\right) \cdot \left(\frac{1-s_0}{2}\right)^{\gamma-1} - s_0^{\gamma} \cdot \left[(0.5+\eta)^{\gamma} + (0.5-\eta)^{\gamma}\right] \quad (2.44)$$

where we have replaced m_1 and m_2 with $0.5+\eta$ and $0.5-\eta$, respectively, to analyze the impact of a more unequal distribution of the elderly population. Differentiating (2.44) immediately yields $\frac{\partial e}{\partial s_0} < 0$ (see the Proof to Proposition 2.7) and $\frac{\partial e}{\partial \eta} < 0$. Differentiating with respect to the crowding parameter α and using the first order condition leads to

$$\begin{aligned}\frac{\partial e}{\partial \alpha} =& 2 \cdot \left(\frac{1-s_0}{2}\right)^{\gamma-1} - \frac{b}{1-b} \cdot s_0^{\gamma} \cdot \\ & \left\{(0.5+\eta)^{\gamma}\left[ln\left(\frac{1-s_0}{2}\right) - ln(s_0) - ln(0.5+\eta)\right] + \right. \\ & \left. (0.5-\eta)^{\gamma}\left[ln\left(\frac{1-s_0}{2}\right) - ln(s_0) - ln(0.5-\eta)\right]\right\}.\end{aligned} \quad (2.45)$$

Numeric evaluation of this expression shows that $\frac{\partial e}{\partial \alpha} > 0$. Hence, we get the following comparative statics results:

$$\frac{ds_0}{d\eta} = -\frac{\partial e/\partial \eta}{\partial e/\partial s_0} < 0 \quad (2.46)$$

and

$$\frac{ds_0}{d\alpha} = -\frac{\partial e/\partial \alpha}{\partial e/\partial s_0} > 0. \quad (2.47)$$

Again, the excessive provision of the publicly provided good for the young is reached earlier, with a lower share of the elderly (s), when the population is more unequally distributed across the municipalities (η) and when the publicly provided good exhibits less crowding (lower α).

10 For the comparative statics, we assume a critical threshold of $s_0 > \frac{1}{2}$.

2.5 Alternative specifications and extensions

The approach chosen is admittedly simple to highlight the conflicting forces of gerontocratic power and fiscal competition. In the following, we will briefly highlight alternative specifications of the model and discuss policy implications for the central government.

Tentative policy implications
One of the messages of our model is that ageing societies may face increasing fiscal competition for the young and mobile population. The fiscal competition may be so strong that a gerontocratic society spends even more on the young population than the social planner. A central government that is aware of this mechanism may try to correct the detrimental effects of fiscal competition. Our model suggests that some centralization of competencies or at least some coordination among municipalities may be beneficial. When ageing has proceeded so far that fiscal competition forces gerontocracies to an excessive provision for the young, an upper level government could improve the allocation of resources by limiting the expenses on some age-specific public services. For instance, the government could provide incentives for the municipalities to coordinate the provision of critical services (such as sports facilities, child care or libraries). Such coordination is beneficial, if the services in neighboring municipalities can be used by commuting (which was excluded from our formal model by assumption). Alternatively the government can make the provision of public services for the young more expensive by reducing matching grants or by limiting the access to subsidized credit programs.

If the autonomy of municipalities prevents such coordination or if the services cannot be accessed by commuting, are there alternative means for an upper level government to alleviate the distortions in the public goods provision? The government could try to attract younger families to areas affected by demographic shrinkage. For instance, it could provide financial incentives for younger people to locate in those areas.[11] In terms of our

[11] We do not claim that such a measure is optimal from a global macroeconomic perspec-

model, this implies an exogenous increase in N. Does an increase in N bring the expenditures for the young closer to the welfare optimum? To analyze this question, we define the budget shares for the publicly provided good of the young in the gerontocratic and welfare scenarios as $BS^G \equiv \frac{2 \cdot Y_i^G}{b \cdot (N+M_1+M_2)}$ and $BS^W \equiv \frac{2 \cdot Y^W}{b \cdot (N+M_1+M_2)}$, respectively. For simplicity, we focus on the case of completely symmetric municipalities (also with respect to the distribution of the elderly, $M_1 = M_2$). If the share of the budget spent on the provision for the young is larger in a gerontocracy than with welfare maximization i.e. $BS^G > BS^W$, then the provision in a gerontocracy is excessive. If

$$BS^G > BS^W \Leftrightarrow \frac{1}{2\alpha} \cdot \frac{1}{1 + DR} > \frac{1}{1 + DR^{\gamma}} \tag{2.48}$$

or $1 - 2 \cdot \alpha - 2 \cdot \alpha \cdot DR + DR^{\gamma} > 0$, where $DR \equiv \frac{M}{N}$ is the dependency ratio, then the provision of the publicly provided goods for the young can be brought closer to the welfare optimum by increasing N and thus decreasing the dependency ratio.

Social planner

We focus on a social planner who governs over an aggregate budget in two different scenarios. Either all young individuals are concentrated in one municipality (Section 2.4.1) or the social planner is bound by an equal provision rule (Section 2.4.3). In principle, our model also allows analyzing alternative welfare scenarios.

An additional conceivable welfare scenario considers one social planner in each municipality. Here, the social planners compete for the young population to maximize the local welfare. This scenario allows the isolation of the impact of gerontocracies as the decision makers differ only in terms of the objective function. However, the analysis is complicated by the critical question of whether the social planner should include or exclude the migrant population in her welfare considerations. Finally, the case of a central gerontocratic government is trivial because no services would be provided for the young.

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tive but solely discuss the measure with respect to the detrimental fiscal competition.

Intergenerational links

Our framework incorporates three dimensions of ageing. Firstly, the model takes into account that ageing causes municipalities to become gerontocracies which changes the power relation. Secondly, we highlight that ageing demands a restructuring of the public goods bundle. Thirdly, ageing also influences the competition for the mobile young. As the young cohorts continuously become smaller there are fewer mobile individuals and consequently the competition intensifies. A fourth facet of ageing that is neglected in our analysis concerns intertemporal linkages. Intertemporal aspects are important for age-specific publicly provided goods. For instance, a gerontocratic society may decide on infrastructure investments suitable for the current population size. This infrastructure, however, will be excessive for the smaller future generations. The current generation thus puts a fiscal burden on future generations and may make the public finances unsustainable. An analysis of such investment decisions requires an intertemporal model, which is beyond the scope of the static model presented here. Accounting explicitly for declining future generations in a fiscal competition framework could bring valuable insights into the consequences of decisions made today. A closer examination of these aspects would certainly be interesting for future work.[12]

2.6 Conclusion

We have developed a simple framework that allows us to analyze the fiscal competition of ageing municipalities. The ageing of a society will lead to shifts in the provision of public services. When ageing advances, it is optimal from a welfare maximizing point of view to gradually substitute publicly provided goods aimed at the young population with publicly provided goods preferred by the elderly population. This substitution process does not only depend on the ageing itself but also on the degree of crowding and on the regional distribution of the elderly population.

[12] The next chapter considers the intergenerational link of long lived publicly provided goods in a setting where future generations are smaller. However, the influence of fiscal competition is not considered.

Once the elderly gain the majority in society, they try to use their power to reduce the provision of the publicly provided good for the young to an inefficiently low level. As our analysis has shown, the downscaling of the provision of the publicly provided good proceeds even slower in a gerontocracy than in a welfare maximizing society. The driving force behind this phenomenon is the fiscal competition for the mobile young among the municipalities. When the share of the elderly is sufficiently large, the utility of the young is higher in gerontocracies than in welfare maximizing societies. Ultimately, the gerontocracies will provide even more of the publicly provided good for the young than the social planner. Put differently: The threat of ageing towns is not so much the exploitation of the young but rather the excessive incentives for making municipalities attractive for the mobile young. The enormous investments in sports facilities and in family-friendly environments which currently occur in the rapidly ageing rural areas in eastern Germany may already be telling examples of this trend. According to a survey of municipalities in the eastern German states of Saxony, Saxony-Anhalt and Thuringia, the preservation and expansion of kindergarten and elementary school infrastructure were stated as the most important aspects of local population policy. Some local decision makers admit that the current supply is intentionally oversized in order to be attractive to families (Sedlacek, 2007).

To our knowledge, this contribution is the first attempt to analyze the consequences of fiscal competition in ageing societies. Our approach is admittedly simple to highlight the main driving forces of this process. Many open questions remain for future research. First, we have mostly focused on the ageing process but demographic change also leads to a declining population size. Second, altruistic motives may change the outcomes, if a significant proportion of the younger generation stays in the municipality of their parents. The elderly may care for the well-being of their children. Such altruism could prevent the downsizing of publicly provided goods for the young, if the young generation remains in the same municipality with their parents. Intergenerational links may also create barriers to mobility for the young, which affects fiscal competition and may even lead to path dependence in the population distribution. Third, the complexity of municipal decision-making goes far beyond the simple provision of two types of publicly provided goods. Alternative means of financing through taxes may change the provision levels of publicly provided goods. Fourth, municipalities do not only compete for households but also for firms. Attracting new investments may create jobs and generate a subsequent inflow of households. Finally, changes in the size of publicly provided goods often entail considerable adjustment costs that have to be taken into account.

Appendix

Appendix 2.A Mixed strategy equilibrium

In the case of two gerontocracies, there is a unique pure strategy equilibrium for $\alpha \geq 1/2$. However, when $\alpha < 1/2$, only a mixed strategy equilibrium exists. In the following, we derive the equilibrium properties of the gerontocratic solution in detail.

The first order condition for municipality i is (see Section 2.3):

$$FOC_i \equiv b \cdot \frac{N}{\left[\left(\frac{Y_j}{Y_i}\right)^{\frac{1}{\alpha}} + 1\right]^2} \cdot \frac{Y_j^{\frac{1}{\alpha}}}{\alpha \cdot Y_i^{\frac{1}{\alpha}+1}} - 1 = 0. \tag{2.49}$$

In order to determine the reaction function, we differentiate the first-order condition with respect to Y_i and Y_j.

$$\frac{\partial FOC}{\partial Y_i} = \frac{bN}{\alpha^2} Y_j^{\frac{1}{\alpha}} \left(\left(\frac{Y_j}{Y_i}\right)^{\frac{1}{\alpha}} + 1\right)^{-3} Y_i^{-\frac{2}{\alpha}-2} \left[(1-\alpha)Y_j^{\frac{1}{\alpha}} - (1+\alpha)Y_i^{\frac{1}{\alpha}}\right] \tag{2.50}$$

and

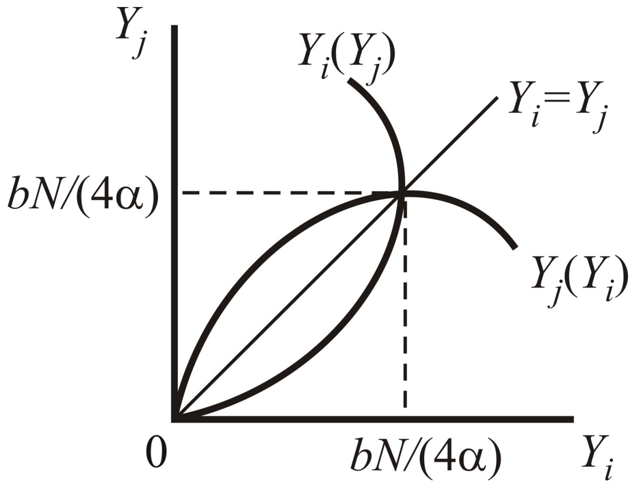
$$\frac{\partial FOC}{\partial Y_j} = \frac{bN}{\alpha^2} Y_i^{-\frac{1}{\alpha}-1} \left(\left(\frac{Y_j}{Y_i}\right)^{\frac{1}{\alpha}} + 1\right)^{-3} Y_j^{\frac{1}{\alpha}-1} \left(-\left(\frac{Y_j}{Y_i}\right)^{\frac{1}{\alpha}} + 1\right). \tag{2.51}$$

Equation (2.50) gives the second-order condition. In equilibrium, we need

$$\frac{\partial FOC}{\partial Y_i} < 0 \Leftrightarrow Y_i > \left(\frac{1-\alpha}{1+\alpha}\right)^{\alpha} Y_j \tag{2.52}$$

for a local maximum. The slope of municipality i's reaction function is given by

Figure 2.3: The pure strategy equilibrium of provision for the young



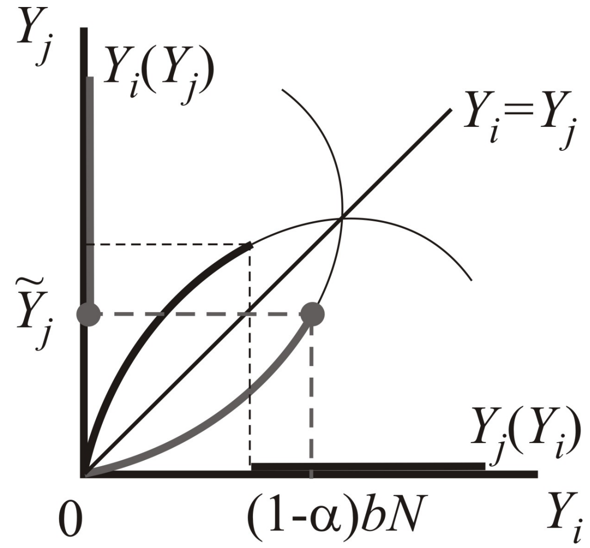
$$\frac{dY_i}{dY_j} = -\frac{\partial FOC/\partial Y_j}{\partial FOC/\partial Y_i} = \frac{Y_i}{Y_j}\frac{Y_i^{\frac{1}{\alpha}} - Y_j^{\frac{1}{\alpha}}}{(1+\alpha)Y_i^{\frac{1}{\alpha}} - (1-\alpha)Y_j^{\frac{1}{\alpha}}}. \tag{2.53}$$

The reaction function has a positive slope for $Y_i < Y_j$ and a negative slope for $Y_i > Y_j$ (as long as the second-order condition is fulfilled). Figure 2.3 illustrates the equilibrium in pure strategies.

There is a unique symmetric equilibrium with $Y_i = Y_j = \frac{bN}{4\alpha}$. So far, we have neglected the possibility that the municipalities are worse off when competing for the young. If a municipality has to spend more on the young than it receives via the per capita grant, it will be better off by withdrawing from the fiscal competition. Hence, in equilibrium, the expenditures on the young must not exceed the grants received for the young ($\frac{bN}{4\alpha} \leq \frac{bN}{2}$), which is the case for $\alpha \geq 1/2$.

For $\alpha < 1/2$, the participation constraints for the municipalities become binding and, therefore, the reaction function exhibits a discontinuity (see Figure 2.4). For low provision levels in municipality j, municipality i competes for the young by providing a positive amount of the publicly provided good. However, when the provision in municipality j exceeds the threshold level $\tilde{Y}_j$, the participation constraint becomes binding for municipality i and it reacts with zero provision for the young. We first determine the threshold $\tilde{Y}_j$ and

Figure 2.4: The mixed strategy equilibrium of provision for the young



then show that the provision of $\tilde{Y_j}$ in one municipality and a randomization of provision in the other municipality is an equilibrium.

The threshold for municipality i is reached when it spends as much on the publicly provided good as it receives in grants for the young: $b \cdot N_i = Y_i$. Substituting the equilibrium migration from (2.5) and solving for Y_j yields

$$Y_j = \left(\frac{bN}{Y_i} - 1\right)^{\alpha} \cdot Y_i. \tag{2.54}$$

This expression determines all provisions of publicly provided goods (Y_i, Y_j) where municipality i is indifferent between providing Y_i and nothing for the young. In addition, the provision of Y_i has to be a local optimum for municipality i. Therefore, we substitute (2.54) into the first-order condition (2.49) and obtain $Y_i = (1-\alpha) \cdot b \cdot N$ and $\tilde{Y_j} = \alpha^{\alpha} \cdot (1-\alpha)^{(1-\alpha)} \cdot b \cdot N$. If municipality j plays $\tilde{Y_j}$, municipality i is just indifferent between $Y_i = (1-\alpha) \cdot b \cdot N$ and $Y_i = 0$.

For an equilibrium, municipality i must choose the alternatives $Y_i = (1-\alpha) \cdot b \cdot N$ with probability p^H and $Y_i = 0$ with probability $1 - p^H$ such that municipality j finds it optimal to play $\tilde{Y_j}$. The expected utility of municipality j amounts to

$$
\begin{aligned}
E\left[U_j(Y_j)\right] =& p^H \left(b(M_j+N_j)-Y_j\right)+(1-p^H)(b(M_j+N)-Y_j)= \\
& bM_j + p^H \frac{bN}{\left(\frac{(1-\alpha)bN}{Y_j}\right)^{1/\alpha}+1} + (1-p^H)bN - Y_j.
\end{aligned} \tag{2.55}
$$

We get the first order condition for an optimal choice of Y_j by differentiating the expected utility:

$$
\frac{\partial E\left[U_j(Y_j)\right]}{\partial Y_j} = p^H \frac{bN}{\left[\left(\frac{(1-\alpha)bN}{Y_j}\right)^{1/\alpha}+1\right]^2} \frac{\left[(1-\alpha)bN\right]^{1/\alpha}}{\alpha Y_j^{1/\alpha+1}} - 1 = 0. \tag{2.56}
$$

Substituting $Y_j = \tilde{Y_j}$ and solving for p^H yields

$$
p^H = \left(\frac{\alpha}{1-\alpha}\right)^{\alpha} < 1. \tag{2.57}
$$

It is easy to verify from the second order condition that $\tilde{Y_j}$ is indeed a maximum $\left[\frac{\partial^2 E[U_j(Y_j)]}{\partial Y_j^2} < 0\right]$. In the mixed strategy equilibrium, municipality i chooses $Y_i = (1-\alpha)\cdot b \cdot N$ with probability $p^H = \left(\frac{\alpha}{1-\alpha}\right)^{\alpha}$ and $Y_i = 0$ with probability $1-p^H$. Municipality j ($j \neq i$) plays $\tilde{Y_j}$.

Chapter 3

Intergenerational publicly provided goods, costly upkeep and population loss

3.1 Introduction

Many economic decisions made today have impacts that last beyond the current generation. However unborn generations affected by current choices cannot vote or sign contracts to protect their interests; this presents a well known political economic conundrum. Some examples of choices that have long lasting impacts include: PAYGO pension schemes, environmental protection, debt and investments in long lived public goods. This chapter focuses on intergenerational public goods (IPGs), i.e. public goods that provide benefits both within the current generation as well as among the successive generation(s). Examples of such public goods include most municipal infrastructure (i.e. roads and parks) (Sandler, 1999). What is the influence of a shrinking population on the provision of locally provided intergenerational public goods?

Local provision of IPG's is particularly affected by changes in the demographic composition. When communities experience population growth new and additional investments in infrastructure (IPG's) is necessary. In times when successive generations are sufficiently large to use and finance the accumulated stock, this growth is sustainable. However, when the number of users declines due to population loss, the ensuing generations may perceive the existing stock as excessive. When population loss is locally confined the continued use of past investments can in part be achieved by consolidating municipalities. However, when through demographic change and emigration vast regions are depopulated, scaling back of existing infrastructure becomes imminent. This may be difficult to accomplish technically and such adjustments may also be costly.

In many developed countries the ongoing demographic transition, will result in more widespread population loss in the next decades. Already today many communities are experiencing population loss due to migration and natural population change. In the future the share of communities experiencing population loss will become larger. Moreover, population loss is not uniform across countries nor between regions within a country. In Table 3.1 the projected population change in Germany is shown for different spatial types for the time period 2008-2025. The developments in the different region types are further differentiated for the eastern and western parts of the country. While the population in the different regions in the western parts will remain relatively stable, the eastern regions will experience marked population loss across all types. The magnitudes in the rural and urbanized areas are large. Of all German counties more than half (225 of 413) are projected to lose population by 2025, some of which will lose up to one quarter of their populations in 2008.[1] For public service provision to be adapted to the changing population, farsighted public policies are necessary. However, past decisions may prevent flexible adjustment.

Local communities provide an array of services whose need is highly dependent on the age composition and population characteristics of the community (schools, child care facilities, recreation and culture). Thus changes in the demographic composition of a community influences the local fiscal burdens (Ladd, 1994; MaCurdy and Nechyba, 2001). Underprovision of publicly

[1] See Table 3.1 and Federal Institute for Research on Building, Urban Affairs and Spatial Development (BBSR) (2010).

Table 3.1: Projected population change 2008-2025 in Germany differentiated by type of region and areas in the east and the west (in %)

Region type	west	east
Agglomerations[a]	0.8	-1.2
Urbanized areas/regional centers[b]	-0.1	-15.2
Rural areas[c]	0.2	-16.4

[a] Population density of 300 persons/km^2 and a total population of more than 300.000.

[b] Either a population density of 150 persons/km^2 or a population of over 100.000 with a density of 100 persons/km^2.

[c] Population density below 100 persons/km^2 if in a region with a center with more than 100.000 inhabitants, otherwise areas with population density below 150 persons/km^2.

Source: Federal Statistical Office and the Federal Institute for Research on Building, Urban Affairs and Spatial Development (BBSR) INKAR (2009)

provided goods arise at the local level when positive spillovers are present (McKinnon and Nechyba, 1997). Such *intra*generational spillovers imply that non-residents from neighboring jurisdictions also benefit from the publicly provided good in another jurisdiction without contributing to its financing. Moreover *inter*generational publicly provided goods may also yield positive spillovers since benefits may incur to ensuing generations. In the presence of intergenerational benefit spillovers, the contribution of an investment made today is underestimated when its longevity and benefit to successive generations if not accounted for. In both cases the publicly provided good will be under provided. Such provision decisions have been studied in a setting of intergenerational clubs, where atemporal and intertemporal decision-making are distinguished (Cornes and Sandler, 1996; Sandler, 1999). Decentralization will lead to more efficient outcomes than centralized provision (Hatfield, 2007) and different tax bases also influence the inefficiency of investment in provision of IPG's (Rangel, 2005; Hatfield, 2007).

This chapter describes the underlying assumptions and the outcomes of a simple model with two generations. In each period only one generation is alive. The single period utility depends on the consumption of a private numeraire good as well as the publicly provided intergenerational good. The period specific consumption of the publicly provided good is influenced by crowding (atemporal) and depreciation (intertemporal). Furthermore, the investments made previously cannot be converted into cash (irreversibility).

The provision levels of a publicly provided intergenerational good are determined for the cases of laissez-faire and welfare maximization. In laissez-faire, the assumed intergenerational link is simply an intergenerational publicly provided good. In the welfare maximizing case, the social planner allocates the aggregated budgets of the two generations. I additionally distinguish between the baseline case and the case of costly maintenance. In the baseline case the benefit the second generation derives from the inherited stock is free of charge. Conversely in the case of costly maintenance, maintenance costs accrue on the inherited investment. Under the respective assumptions the allocation choices differ.

In the baseline case with generational laissez-faire, when the first generation becomes larger, the utility of this generation will increase due to economies of scale in the public goods provision. When the second generation grows this is not necessarily the case. Instead for the second generation there is a critical level of population loss beyond which no publicly provided goods are provided. If the second generation does not invest in publicly provided goods, then the utility of the second generation declines in response to a size increase. Moreover, if public provision does take place, an increase in the size of the second generation may still lower the utility of this generation since the disadvantage of sharing among more users may dominate the benefit of allocating more resources.

In comparison to the provision with a social planner, the first generation under-invests in public provision in laissez-faire. Furthermore, the public provision and consumption levels by the second generation in the social planner's case are either less than or equal to the corresponding levels in laissez-faire. In the event that the second generation does not unconditionally benefit from the investments made in the past, the allocation choices change. If costs of maintenance accrue on the inherited investment, then the effect of increasing provision in the first period is no longer unambiguously positive. In contrast to the baseline case, the provision in the first period may be lower in welfare maximization than laissez-faire, if the costs of maintaining the inherited stock is relatively high. If population loss is severe and the inherited stock as well as the cost of maintenance is large, then default (in the sense of no private consumption) may occur in laissez-faire.

This chapter is structured as follows: the second section presents the model setup. The third section derives the solutions for the baseline case in the laissez-faire setting and additionally considers the effect of increasing wealth. In the fourth section the intergenerational welfare maximization is derived and the welfare maximizing results are compared to the laissez-faire outcomes. Similarly in sections five and six the influence of costly maintenance is considered in the cases of laissez-faire and welfare maximization. Additionally the results are compared to the baseline case. The final section concludes.

3.2 The model

The model descirbes the provision of publicly provided goods at the local level over a timespan of two generations.[2] The two generations do not overlap, instead an intergenerational trade-off is generated over the intergenerational characteristic of the publicly provided good. When the investment made by one generation is long lasting and subsequently inherited (at least partially), then the remaining stock of publicly provided good from this generation influences the choice of consumption and investments made by the ensuing cohort.

3.2.1 Preferences of the two generations

The preferences of the representative individual are described by a utility function over the consumption of a private good c_i and a function of a publicly provided good $f(G_i)$ where $i = 1, 2$. For the first generation the utility is

[2] The term generation is used for a period of many years that is relatively long in comparison to election periods. A familial generation, or the mean interval between successive generations (the mean age of mothers in a given year), is approximately 30 years (van de Walle, 1982).

given by:

$$U_1(c_1, f(G_1)) \qquad where \qquad f(G_1) = \left(\frac{G_1}{N_1^{\alpha}}\right). \tag{3.1}$$

The function f describes the congestion by usage common to locally provided public goods. N_1 is the size of the first generation and the parameter $\alpha \in [0,1]$ determines the degree of congestion. At the extremes, when $\alpha = 0$ or $\alpha = 1$, the characteristic of the publicly provided good is respectively either purely public or private. The empirically relevant intermediate levels of crowding of locally provided goods lie within the range $0 < \alpha < 1$ (Borcherding and Deacon, 1972; Bergstrom and Goodman, 1973; Edwards, 1990).

The utility function of the second generation is given by:

$$U_2(c_2, g(G_1, G_2)) \qquad where \qquad g(G_1, G_2) = \left(\frac{\delta G_1 + G_2}{N_2^{\alpha}}\right), \tag{3.2}$$

where the function g not only captures the congestion of the use of the publicly provided good, but also includes the intergenerational spillover to the second generation from the investments made by the previous generation. The congestion from usage by the number of members of the second generation is analogous to that of the first generation, and is given by N_2^{α}. The depreciation of the publicly provided good is $\delta \in [0,1]$.[3] When $\delta = 0$ the publicly provided good is not intergenerational, in the sense that none of the investment made by the first generation will be available to the members of the second generation. When $\delta = 1$ all of what was invested in the first period will also be available to the second generation. In general, a higher δ implies more of the benefit is carried over. As will be shown below, very low rates of depreciation may become "problematic" when a municipality is experiencing population loss. I assume that it is always possible for the second generation to make additional investments in the publicly provided good ($G_2 \geq 0$), but I do not allow $G_2 < 0$.

[3] For the treatment of intergenerational spillovers a similar specification is used by Hatfield (2007). However, his analysis is restricted to additively separable utility functions.

3.2.2 Budget constraints

The budget constraint for a member of each generation ($i = 1, 2$) is defined by an exogenously given level of wealth w_i which is equated to the expenditures on consumption and a head tax t_i. This tax reflects the individual's contribution to the production of the publicly provided good, and the price of the private good is normalized to unity,

$$w_i = c_i + t_i. \tag{3.3}$$

The tax has to be $t_i = \frac{\theta_i G_i}{N_i}$, for the government budget to equate to $t_i N_i = \theta_i G_i$. The production of the publicly provided good is linear, such that θ units of private good produces one unit of the publicly provided good (since there may be technological progress, the θ's of the two periods do not need to be equal). The total aggregate budget of generation i is given by:

$$N_i w_i = N_i c_i + \theta_i G_i. \tag{3.4}$$

For ease of exposition the subscripts are dropped whenever a parameter or variable is assumed to remain constant between generations. Furthermore, no discounting is assumed.

The above setup describes a very simple framework to study intergenerational interdependence of two generations. To analyze the effects of a change in the population on the provision of the publicly provided good, the size of the second generation N_2 is assumed to be different from that of the first generation ($N_1 \neq N_2$). This general model framework not only allows the analysis of population loss, but results are also derived for different assumptions of economic growth (in terms of an increase in the exogenous wealth) and in Section 3.5 the possibility of costly upkeep is considered. The results from the case of two generations acting under laissez-faire assumptions is contrasted with those achieved under a benevolent (foresighted) social planner.

3.3 Laissez-faire generations: the baseline case

To reach explicit solutions for the consumption and provision levels of the publicly provided good a specific functional form is chosen for the utility functions. Here Cobb-Douglas utility functions are assumed. In order to solve for the laissez-faire solution, the utility of each generation will be maximized independently. First the solutions to the first generation problem are derived. Subsequently the allocations of the second generation, which are dependent on the provision chosen by the first generation, are solved for.

First period utility The utility function of the representative member of the first period defines the choice between private consumption (c_1) and the publicly provided good G_1:

$$\begin{aligned} \max_{c_1, G_1} \quad & U_1 = c_1^{\beta} \left(\frac{G_1}{N_1^{\alpha}} \right)^{1-\beta} \\ s.t. \quad & w = c_1 + \frac{\theta_1 G_1}{N_1}. \end{aligned} \tag{3.5}$$

The crowding parameter is $\alpha \in [0, 1]$. As described in Section 3.2, this means that the amount of publicly provided good enjoyed by the representative member depends on the level of crowding. $\beta \in (0, 1)$ to ensure diminishing marginal utility. The preference between public and private consumption is given by the parameter β. In period 1 the resulting allocations are the following:

$$G_1^L = \frac{(1-\beta) w N_1}{\theta}, \tag{3.6}$$

$$c_1^L = \beta w, \tag{3.7}$$

where the superscript L identifies the allocations in the case of laissez-faire. The provision of the publicly provided good G_1^L depends positively on the wealth, w, and the first period population N_1. Private consumption is obviously independent of the size of the population.

Second period utility The preferences of the second generation are similar. However, the depreciated stock of the publicly provided good chosen by the first generation additionally enters the utility function. This signifies an impure intergenerational spillover. The utility function of the representative member of the second period thus defines the choice between private consumption (c_2) and the publicly provided good (as the sum of δG_1 and G_2). Specifically the utility maximization problem of the second generation is:

$$\begin{aligned} \max_{c_2, G_2} \quad & U_2 = c_2^{\beta} \left(\frac{\delta G_1 + G_2}{N_2^{\alpha}} \right)^{1-\beta} \\ s.t. \quad & w = c_2 + \frac{\theta_2 G_2}{N_2}. \end{aligned} \tag{3.8}$$

The effective level of publicly provided good depends on the amount of depreciated public provision from the first generation δG_1 (where $\delta \in [0, 1]$, and $1 - \delta$ is the rate of depreciation and G_1 stands for the provision level) and the current provision G_2.

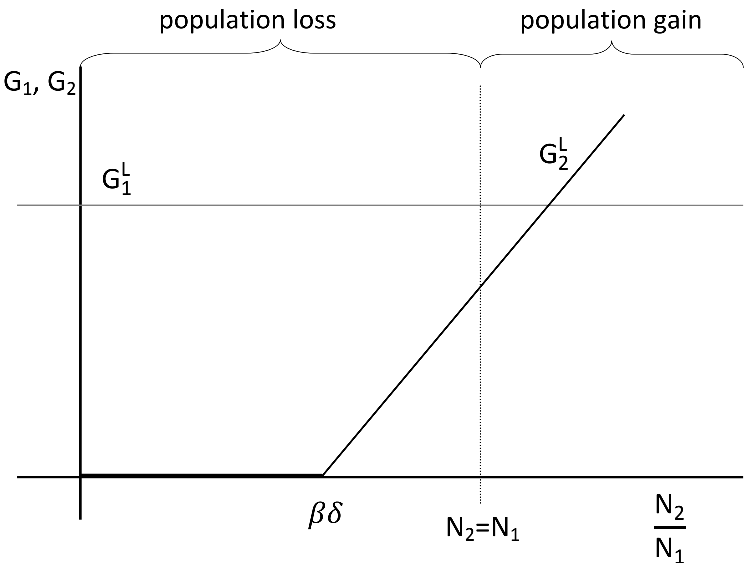
In other words, from the view of the second generation, only the total stock of publicly provided goods are relevant. Thus δG_1 and G_2 are perfect substitutes. Using the budget constraint yields the following provision level of G_2 in period two in terms of G_1:

$$G_2(G_1) = \max \left\{ \frac{(1-\beta) w N_2}{\theta} - \beta \delta G_1; 0 \right\}. \tag{3.9}$$

The above expression includes the intergenerational spillover: δG_1. The level of G_2 is declining in G_1 and is rising in the wealth w and the size of the second generation population N_2. There is incomplete crowding out: when more of G_1 is inherited, the level of investment in G_2 will be lower.[4] In order to find the consumption and provision levels in the second period in terms of the population variables, the level of G_1 from the first period problem (equation (3.6)) is used. The provision level of G_2^L is written in the below form since it may be optimal from the view of the second generation not to provide any publicly provided goods at all. Or conversely the existing stock of publicly

[4] Specifically, a one unit decrease in G_1 leads to a less than one unit increase in G_2. $0 < |\frac{dG_2}{dG_1}| < 1$ since $0 < |-\beta\delta| < 1$.

Figure 3.1: Provision levels of the intergenerational publicly provided goods G_1 and G_2 in the baseline case, plotted against the population relation $\frac{N_2}{N_1}$



provided goods will benefit the successive generation but may be insufficient and require additional new investments.[5]

$$G_2^L = \begin{cases} 0 & \text{if } \frac{N_2}{N_1} \leq \beta\delta, \\ (1-\beta)\frac{w}{\theta}\left(N_2 - \beta\delta N_1\right) & \text{if } \frac{N_2}{N_1} > \beta\delta. \end{cases} \tag{3.10}$$

Proposition 3.1. *If the population shrinks moderately or grows such that* $\frac{N_2}{N_1} > \beta\delta$*, then the level of* G_2^L *will be greater than zero. If the population shrinks such that* $\frac{N_2}{N_1} \leq \beta\delta$ *then* $G_2^L = 0$*.*

Proof. Follows from equation (3.10) and is illustrated in Figure 3.1.

5 Some authors have also considered the cost of population growth to local communities. Ladd (1994) examines the relationship between population growth and per capita spending. She argues that in theory population growth could either cause an increase in spending and service quality, or an effective decrease in spending. For instance, population growth that increases population density, may on the one hand make infrastructure more effective and hence reduce per capita spending. On the other hand, increased population density may create a "harsher environment" (in terms of public safety and fire protection) in which case service provision is more costly. Hence the effect of population growth on per capita spending is a priori unclear.

In Figure 3.1, G_1^L is independent of N_2 and only for $\frac{N_2}{N_1} > \beta\delta$ is the provision $G_2^L > 0$. If $G_2^L = 0$ the existing stock of G_1^L is more than necessary to maximize the utility of the second generation given the budget constraint. In this case, private consumption could be further expanded if the existing stock could be converted into cash. Whether it is possible to scale back the existing stock at no cost depends on the publicly provided good. For instance, in the case of school buildings the municipality may be able to sell the building to a private investor and convert the profits into lower taxes. However, not all investments are as easily convertible as a building, for example a four lane street is not easily transformed into a two lane street when demand is halved. If it is not possible to scale back existing stock, the second generation would not fully benefit from the choices made by the members of the first generation.

The above public provision levels are subsequently used to solve for the levels of private consumption. Private consumption levels depend on the size of the second generation. If the depreciated stock of G_1^L is sufficiently high and the population loss is severe then no G_2^L may be provided, in which case, all wealth will be allocated toward private consumption. The expressions for private consumption are:

$$c_2^L = \begin{cases} w & \text{if } \frac{N_2}{N_1} \leq \beta\delta, \\ \frac{\beta w}{N_2}\left(N_2 + \delta(1-\beta)N_1\right) & \text{if } \frac{N_2}{N_1} > \beta\delta. \end{cases} \tag{3.11}$$

Consequently, using the derived public provision and private consumption levels I find the generation specific utility functions (U_1 and U_2) in terms of the population variables:

$$U_1^L = \left(\frac{1-\beta}{\theta}\right)^{1-\beta} \beta^\beta N_1^{1+\beta(\alpha-1)-\alpha} w \tag{3.12}$$

and

$$U_2^L = \begin{cases} \left(\frac{(1-\beta)\delta N_1}{\theta}\right)^{1-\beta} N_2^{\alpha(\beta-1)} w & \text{if } N_2 \leq \beta\delta N_1, \\ \left(\frac{1-\beta}{\theta}\right)^{1-\beta} \beta^\beta N_2^{\beta(\alpha-1)-\alpha} w \left[N_2 + N_1\delta(1-\beta)\right] & \text{if } N_2 > \beta\delta N_1. \end{cases} \tag{3.13}$$

The effect of a change in the size of the population of the first generation N_1 on the utility levels of the first and second generations are positive: $\frac{\partial U_1^L}{\partial N_1} > 0$ and $\frac{\partial U_2^L}{\partial N_1} > 0$. If the size of the population in the first generation increases, then the amount of publicly provided good will increase. Since $\alpha < 1$ this will have a positive effect on the utility of both generations.

When the size of the second generation changes the effect on the utility of the second generation depends on whether any public provision takes place in the first place ($G_2^L \geq 0$). If $N_2 \leq \beta\delta N_1$ and thus $G_2^L = 0$, then $\frac{\partial U_2}{\partial N_2} < 0$. An increase in the size of the second generation population implies that the inherited publicly provided good is shared among more users, and hence causes a decline in the utility. However, if $N_2 > \beta\delta N_1$ the effect of a change in the size of the second generation on the utility of the second generation is U-shaped. The benefit of more resources may not be realized, if the additional users crowd the benefits of the public goods provision.

Lemma 3.1. *If $N_2 > \beta\delta N_1$, the second generation utility function is convex with respect to changes in the size of this generation.*

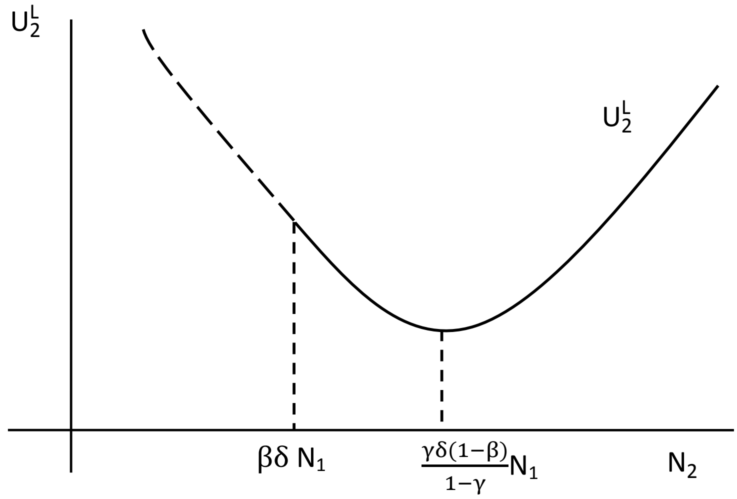
The utility of the second generation first decreases and then increases when the size of this generation increases (given $G_2^L > 0$). The shape of the utility function with respect to the size of the second generation is illustrated in Figure 3.2.

The form of the function results from the dependence of the second period utility on the provision in the first period and the relative size of the two generations as well as the characteristics of the publicly provided good. The first-order condition (FOC) of U_2^L with respect to N_2 is:

$$\frac{\partial U_2^L}{\partial N_2} = \left(\frac{1-\beta}{\theta}\right)^{1-\beta} \beta^\beta w N_2^{-\gamma} \left[1 - \gamma\left(1 + \frac{N_1}{N_2}\delta(1-\beta)\right)\right] \gtreqless 0 \quad \text{for} \quad N_2 > \beta\delta N_1 \tag{3.14}$$

where $\gamma \equiv \beta(1-\alpha) + \alpha$. At $N_2 = N_1\beta\delta$ the second generation utility is declining in N_2, $\frac{\partial U_2}{\partial N_2} < 0$. Furthermore, equation (3.14) is at an extremum when the expression in brackets is zero, i.e. if $\left[1 - \gamma\left(1 + \frac{N_1}{N_2}\delta(1-\beta)\right)\right] = 0$. Solving gives: $N_2 = \frac{\gamma\delta(1-\beta)}{1-\gamma}N_1$. I verify that this point is a minimum by differentiating the above first order-condition (in equation (3.14)) to give the

Figure 3.2: The second generation utility function is U-shaped with respect to the size of the second generation (N_2)



second-order condition: $\frac{\partial FOC}{\partial N_2} > 0$. Thus for a second generation population $\beta\delta N_1 < N_2 < \frac{\gamma\delta(1-\beta)}{1-\gamma}N_1$ the utility of the second generation will decline for an increase in the size of the population. In other words, in this range the disadvantage of sharing dominates the benefit of more resources. When $N_2 > \frac{\gamma\delta(1-\beta)}{1-\gamma}N_1$, a marginal increase in the size of the second generation will cause the utility of this generation to increase.

The critical size of the second generation depends on the counteracting effects of crowding and depreciation. In particular if crowding increases ($\alpha \to 1$), if depreciation increases ($\delta \to 0$) and if the population shrinks ($N_2 \downarrow$); then the minimum will move to the right. The utility of the second generation will decline for a larger second generation population. Therefore, in contrast to the first generation utility, the utility of the second generation may decline when the number of users rises.

The above section describes the baseline model of generational laissez-faire. The only intergenerational link is the (depreciated) intergenerational public good. However, other linkages, which may attenuate or strengthen the effects of population loss, are also conceivable. One such alternative specification, namely increasing wealth, is considered in the following section. Section 3.5 is devoted to a thorough discussion of the effect of introducing a cost on the usage of the inherited stock on the second generation. These results are subsequently contrasted with the above baseline case outcomes.

Excursion: The effect of increasing wealth

So far I assumed that the endowment per person is equal between the two periods. Given economic growth, this need not necessarily be the case. More wealthy members of a community may demand more (street-lights for previously unlit streets) or better services (for example in schools or kindergartens). To study any potential wealth effects I assume in the following section that $w_2 > w_1$, implying that the second generation is wealthier. How does increasing wealth influence the provision of the publicly provided good?

Using the altered budget constraints, the allocations are as follows:

$$G_1^w = \frac{(1-\beta)w_1 N_1}{\theta}$$
$$c_1^w = \beta w_1$$

and

$$G_2^w = \begin{cases} 0 & \text{if} \quad \frac{N_2}{N_1} \leq \beta\delta\frac{w_1}{w_2}, \\ \frac{(1-\beta)}{\theta}\left(w_2 N_2 - \beta\delta w_1 N_1\right) & \text{if} \quad \frac{N_2}{N_1} > \beta\delta\frac{w_1}{w_2}. \end{cases}$$

$$c_2^w = \begin{cases} \beta\left(w_2 + \delta(1-\beta)w_1\frac{N_1}{N_2}\right) & \text{if} \quad \frac{N_2}{N_1} \leq \beta\delta\frac{w_1}{w_2}, \\ w_2 & \text{if} \quad \frac{N_2}{N_1} > \beta\delta\frac{w_1}{w_2}, \end{cases}$$

when w_1 and w_2 are appropriately substituted in for w in the expressions (3.6), (3.7), (3.10) and (3.11). The superscript w distinguishes these expressions from the previous section. The demands for the private as well as the publicly provided goods are increasing in wealth. The interpretation of the expressions for the first generation remain unchanged. However, in the expressions of the second generation, the effect of wealth is now visible. $G_2^w > 0$

if $w_2N_2 > \beta\delta w_1N_1$, even if there is population loss, such that $N_2 < N_1$, the assumed increase in wealth will offset some of the effect. Since rising income implies an increased demand for the publicly provided good, a faster rate of population loss is thus required to reach the critical level of population loss beyond which no resources will be invested in G_2^w.

3.4 Welfare maximization with an intertemporal budget: the baseline case

The previous sections show that due to the simple intergenerational spillover of publicly provided goods, the provision choices of the subsequent generation are altered. Furthermore the second generation can benefit from an inherited stock since no costs are associated with its usage. In the following section, the welfare maximizing outcomes are computed, and subsequently these are contrasted with the results obtained in the baseline laissez-faire case.

Suppose now that a foresighted benevolent social planner allocates the current and next generation resources toward the provision for both generations. Thus the total budget of the two generations is available in the first period and the first generation utility is maximized under the condition that the second generation utility attains some level of utility $\bar{U_2}$. Analogously to the laissez-faire case, I assume no discounting. The individual generational utility functions remain the same but the aggregate intertemporal budget constraint now reads:

$$(N_1 + N_2)w = c_1N_1 + c_2N_2 + \theta(G_1 + G_2). \tag{3.15}$$

The welfare of the first generation is maximized subject to the above budget constraint and the aforementioned constraint on the second generation utility:

$$\max_{c_1,c_2,G_1,G_2} \quad W^{SP} = c_1^{\beta}\left(\frac{G_1}{N_1^{\alpha}}\right)^{1-\beta} \tag{3.16}$$
$$\text{s.t.} \quad w(N_1+N_2) = c_1N_1 + c_2N_2 + \theta(G_1+G_2)$$
$$\bar{U}_2 = c_2^{\beta}\left(\frac{\delta G_1 + G_2}{N_2^{\alpha}}\right)^{1-\beta}.$$

There are infinitely many conceivable $\bar{U}_2$'s. However, to compare the outcomes of the baseline laissez-faire case to those under welfare maximization, I equate the exogenous level of reservation utility ($\bar{U}_2$) to the level achieved from utility maximization in laissez-faire (U_2^L in expression (3.13)). In other words, to facilitate a welfare analysis I focus in the following on the specific case where $\bar{U}_2 = U_2^L$. Put differently: I ask whether U_1 can be increased while keeping U_2 constant. The results for a generic $\bar{U}_2$ can be found in the appendix in Section 3.A.1.

3.4.1 Provision levels of the publicly provided goods

Solving the above maximization problem and using U_2^L gives the following expression for the provision toward the first generation under welfare maximization (where SP stands for the case of the social planner):

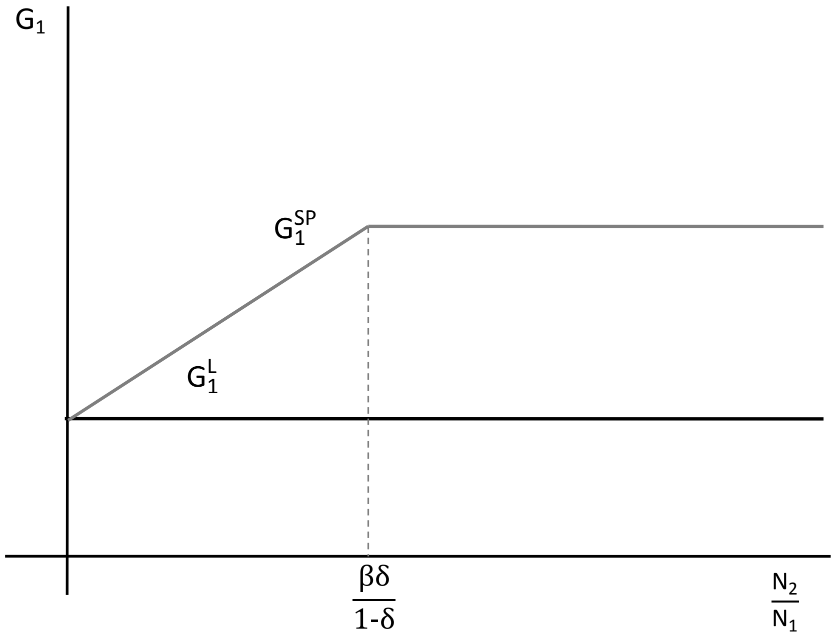
$$G_1^{SP} = \begin{cases} \left(\frac{1-\beta}{\theta}\right) w(N_1+N_2) & \text{if } \frac{N_2}{N_1} \leq \frac{\beta\delta}{1-\delta}, \\ \left(\frac{1-\beta}{\theta}\right) wN_1\left[1+\frac{\delta\beta}{1-\delta}\right] & \text{if } \frac{N_2}{N_1} > \frac{\beta\delta}{1-\delta}. \end{cases} \tag{3.17}$$

The ranges for the two cases are slightly different than in the baseline laissez-faire case. When $\frac{N_2}{N_1} \leq \frac{\beta\delta}{1-\delta}$ the provision of G_1^{SP} now depends on the population in both generations, and is rising in the size of both ($\frac{\partial G_1^{SP}}{\partial N_1} > 0$, $\frac{\partial G_1^{SP}}{\partial N_2} > 0$). When $\frac{N_2}{N_1} > \frac{\beta\delta}{1-\delta}$, G_1^{SP} is independent of N_2. These results are illustrated in Figure 3.3.

Analogously the expressions for G_2^{SP} are:

$$G_2^{SP} = \begin{cases} 0 & \text{if } \frac{N_2}{N_1} \leq \frac{\beta\delta}{1-\delta}, \\ \left(\frac{1-\beta}{\theta}\right) w\left[N_2 - N_1\frac{\beta\delta}{1-\delta}\right] & \text{if } \frac{N_2}{N_1} > \frac{\beta\delta}{1-\delta}. \end{cases} \tag{3.18}$$

Figure 3.3: Comparison of the second generation provision levels in laissez-faire G_1^L and welfare maximization G_1^{SP}, plotted against the population relation $\frac{N_2}{N_1}$

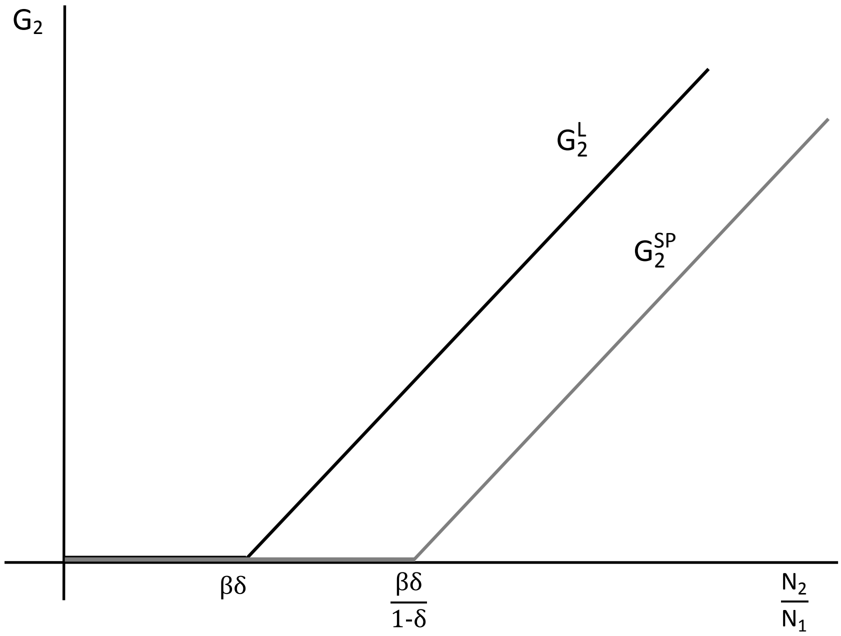


As in the laissez-faire case, for $\frac{N_2}{N_1} \leq \frac{\beta\delta}{1-\delta}$ the provision of G_2^{SP} is zero. Moreover, from the second line of the above expression it is clear that $G_2^{SP} > 0$ only for $\frac{N_2}{N_1} > \frac{\beta\delta}{1-\delta}$. Since $1-\delta < 1$, positive provision of G_2^{SP} will be attained for a larger population relation than for G_2^L. This is illustrated in Figure 3.4 and the result is summarized in:

Proposition 3.2. *When the second generation utility in the case of laissez-faire and a social planner are equalized:*
a) Public provision for the first generation in the social planner's case is higher than the provision in laissez-faire ($G_1^{SP} > G_1^L$).
b) Public provision for the second generation in the social planner's case is less than than or equal to the provision in laissez-faire ($G_2^{SP} \leq G_2^L$).

Proof: a) $G_1^{SP} > G_1^L$ since when $\frac{N_2}{N_1} \leq \frac{\beta\delta}{1-\delta}$ the comparison of functions (3.6) and (3.17) gives: $N_1 + N_2 > N_1$. Furthermore, when $\frac{N_2}{N_1} > \frac{\beta\delta}{1-\delta}$ the comparison of welfare maximization to laissez-faire is: $\frac{\beta\delta}{1-\delta} > 0$. See also Figure 3.3.
b) See above paragraph and Figure 3.4.

Figure 3.4: Comparison of the second generation provision levels in laissez-faire G_2^L and welfare maximization G_2^{SP}, plotted against the population relation $\frac{N_2}{N_1}$

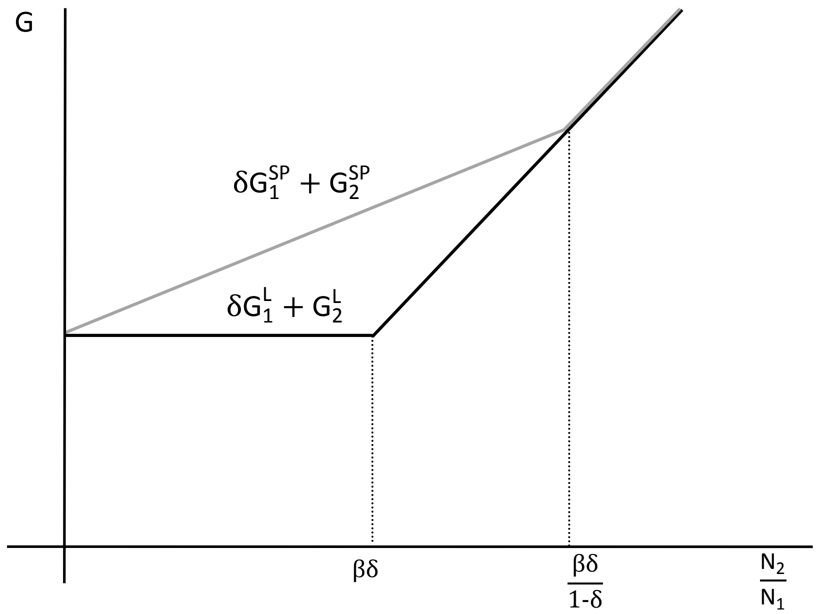


Since in welfare maximization the critical population size beyond which G_2^{SP} will be zero is: $\frac{N_2}{N_1} = \frac{\beta\delta}{1-\delta}$ this level is higher than in the case of laissez-faire (where $\frac{N_2}{N_1} = \beta\delta$). In the case of laissez-faire, provision of G_2^L will be positive for shrinking populations ($\frac{N_2}{N_1} < 1$). However, in the case of the social planner, if $\frac{\beta\delta}{1-\delta} > 1$ then positive provision of G_2^{SP} will require population growth ($\frac{N_2}{N_1} > 1$).[6] In order for the utility maximizing level to be achieved by the second generation under welfare maximization, the social planner will invest more in G_1 and less in G_2 than would be the case under laissez-faire. Thus from a welfare maximizing point of view too much G_2^L is provided under laissez-faire whereas G_1^L is under-provided.

What is the aggregate public provision level $G = \delta G_1 + G_2$ in the laissez-faire and welfare maximizing cases? If $\frac{N_2}{N_1} > \frac{\beta\delta}{1-\delta}$ then the aggregate provisions

6 The population relation $\frac{N_2}{N_1} = \frac{\beta\delta}{1-\delta}$ determines the point at which provision for the second generation will be positive under the social planner. This point is equal to one when $\beta = \delta = .618$. If β or $\delta < .618$ then $\frac{N_2}{N_1} < 1$. If β or $\delta > .618$ then $\frac{N_2}{N_1} > 1$.

Figure 3.5: Comparison of the aggregate provision of intergenerational publicly provided goods $G = \delta G_1 + G_2$ in laissez-faire and the social planner's case



will be equal. If however, $\frac{N_2}{N_1} < \frac{\beta\delta}{1-\delta}$ then $G^{SP} > G^L$. The aggregate public goods levels are illustrated in Figure 3.5. Thus the total provision will be higher under the social planner than under laissez-faire to the point when provision for both generations is positive also in the welfare maximizing case.[7]

3.4.2 Private consumption

The private consumption achieved by the first generation in welfare maximization when the second generation utilities are equalized ($\bar{U_2} = U_2^L$) are also influenced by the population relations. The relevant population ranges are determined by the public goods provision for the second generation and the utility levels that result. Since, as shown above, the threshold populations for positive provision are not the same in laissez-faire and the social

[7] For the first generation the difference in provision is given by $G_1^{SP} > G_1^L \Leftrightarrow \frac{1-\delta(1-\beta)}{1-\delta} > 1$. The difference in the provision of the second generation is given by $G_2^{SP} < G_2^L \Leftrightarrow -\frac{1}{1-\delta} < -1$.

planner's case, three ranges arise for private consumption.[8] Consumption of the first generation is given by:

$$c_1^{SP} = \begin{cases} w\left(1+\frac{N_2}{N_1}\right)\left[\beta - \frac{N_2}{N_1}\left(\frac{N_1}{N_1+N_2}\right)^{\frac{1}{\beta}}\right] & \text{if } \frac{N_2}{N_1} \leq \beta\delta, \\ \beta w\left(1+\frac{N_2}{N_1}\right)\left[1-\delta\left(\frac{N_2+N_1\delta(1-\beta)}{\delta(N_1+N_2)}\right)^{\frac{1}{\beta}}\right] & \text{if } \beta\delta < \frac{N_2}{N_1} \leq \frac{\beta\delta}{1-\delta} \\ \beta w\left(1-\delta(1-\beta)\right) & \text{if } \frac{N_2}{N_1} > \frac{\beta\delta}{1-\delta}. \end{cases} \quad (3.19)$$

These levels are in turn compared to the consumption in laissez-faire where $c_1^L = \beta w$ (see equation: (3.7)). Whether $c_1^{SP} \gtreqless c_1^L$ depends on the relevant population range. The function c_1^{SP} is non-linear in the population relation within this range. In Figure 3.6 the different levels of private consumption of generation one in laissez-faire and welfare maximization are drawn. When the population of generation two approaches zero, the private consumption of the first generation in laissez-faire and in the social planner's case are identical. When $\frac{N_2}{N_1} > \frac{\beta\delta}{1-\delta}$ the consumption by the first generation is constant with respect to the population relation but remains lower in welfare maximization than in laissez-faire ($c_1^{SP} < c_1^L \Rightarrow 1-\delta(1-\beta) < 1$).

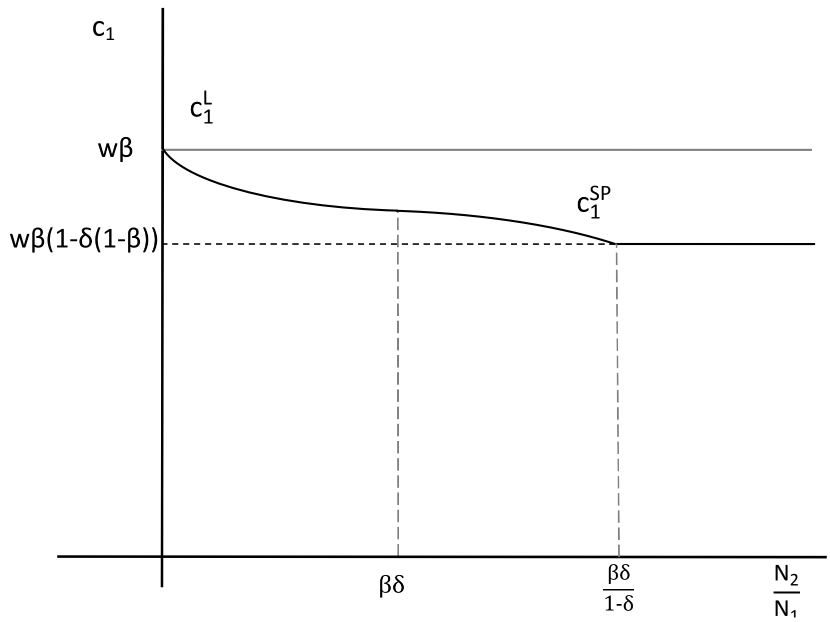
Similarly the comparison with respect to the second generation consumption levels is given by expression (3.11) and:

$$c_2^{SP} = \begin{cases} w\left(\frac{N_1}{N_1+N_2}\right)^{\frac{1-\beta}{\beta}} & \text{if } \frac{N_2}{N_1} \leq \beta\delta, \\ \beta\delta w\left(1+\frac{N_1}{N_2}\right)\left[\frac{N_2+N_1\delta(1-\beta)}{\delta(N_1+N_2)}\right]^{\frac{1}{\beta}} & \text{if } \beta\delta < \frac{N_2}{N_1} \leq \frac{\beta\delta}{1-\delta}, \\ \frac{\beta w}{N_2}\left[N_2+\delta(1-\beta)N_1\right] & \text{if } \frac{N_2}{N_1} > \frac{\beta\delta}{1-\delta}. \end{cases} \quad (3.20)$$

When $\frac{N_2}{N_1} > \frac{\beta\delta}{1-\delta}$ the private consumption by the second generation will be equal in laissez-faire and welfare maximization ($c_2^L = c_2^{SP}$). However, when

[8] First, in the range $\frac{N_2}{N_1} \leq \beta\delta$ the utility comparison is for the cases when public provision for the second generation is zero in both laissez-faire and welfare maximization. Second, in the range $\beta\delta < \frac{N_2}{N_1} \leq \frac{\beta\delta}{1-\delta}$ public provision for the second generation is positive in laissez-faire but zero in welfare maximization ($\bar{U}_2$ is now equated to the lower line of U_2^L). Third, public provision for the second generation is positive in both laissez-faire and welfare maximization which is again reflected in the equated utilities.

Figure 3.6: Comparison private consumption levels c_1^L and c_1^{SP} in laissez-faire and the social planner's case when $\bar{U}_2 = U_2^L$



$\frac{N_2}{N_1} \leq \beta\delta$ the private consumption under laissez-faire is equal to the whole wealth since none is spent on public provision. In this case the consumption under the social planner will be lower $c_2^{SP} < c_2^L$.

3.4.3 First generation utility

The second generation utility in laissez-faire and the social planner's case are by definition set equal, however the utility achieved by the first generation will be different. In the laissez-faire case the utility levels will be lower than with a social planner. In the social planner's case the endowment of the second generation can be allocated already by the first generation, thus a resource transfer from the second generation to the first can take place. In laissez-faire this is not the case.

In the welfare maximizing case the utility achieved by the first generation is described by:

$$U_1^{SP} = \begin{cases} N_1^{-\gamma} w(N_1+N_2)\left(\frac{1-\beta}{\theta}\right)^{1-\beta}\left[\beta - \frac{N_2}{N_1}\left(\frac{N_1}{N_1+N_2}\right)^{\frac{1}{\beta}}\right]^{\beta} & \text{if } \frac{N_2}{N_1} \leq \beta\delta, \\ N_1^{-\gamma} w(N_1+N_2)\left(\frac{1-\beta}{\theta}\right)^{1-\beta}\left[\beta - \beta\delta\left(\frac{N_2+N_1\delta(1-\beta)}{\delta(N_1+N_2)}\right)^{\frac{1}{\beta}}\right]^{\beta} & \text{if } \beta\delta < \frac{N_2}{N_1} \leq \frac{\beta\delta}{1-\delta}, \\ N_1^{1-\gamma}(1-\delta)^{\beta-1}\left(\frac{1-\beta}{\theta}\right)^{1-\beta}\beta^{\beta} w\left[1-\delta(1-\beta)\right] & \text{if } \frac{N_2}{N_1} > \frac{\beta\delta}{1-\delta}. \end{cases} \tag{3.21}$$

The relationship between U_1^L and U_1^{SP} depends on the three ranges of the population relations (whether $\frac{N_2}{N_1} < \beta\delta$, $\beta\delta < \frac{N_2}{N_1} \leq \frac{\beta\delta}{1-\delta}$ or $\frac{N_2}{N_1} > \frac{\beta\delta}{1-\delta}$).

Lemma 3.2. *The utility of the first generation is always higher in the social planner's case than in laissez-faire.*
a) When (in laissez-faire and with the social planner) no additional publicly provided goods are provided for the second generation the welfare loss decreases with the population.
b) When provision for the second generation does take place, the welfare differential between laissez-faire and the social planner is constant with respect to the population.

Comparing the above expressions to the utility achieved under laissez-faire from Section 3.3 equation (3.12) for the case when $\frac{N_2}{N_1} \leq \beta\delta$ gives:

$$U_1^{SP} > U_1^L \quad \Leftrightarrow \quad 1 + \frac{N_2}{N_1} > \beta\left(1 + \frac{N_2}{N_1}\right)^{\frac{1}{\beta}}.$$

Hence the utility differential depends on the relative size of the second generation. When there is no public provision for the second generation, the smaller N_2 is in relation to N_1 the larger the utility differential in favor of welfare maximization will be. For permissible parameter values, the utility achieved under laissez-faire will always be lower than under the social planner.

When $\beta\delta < \frac{N_2}{N_1} \leq \frac{\beta\delta}{1-\delta}$ the utility of the first generation with the social planner is compared to that in laissez-faire according to:

$$U_1^{SP} > U_1^L \quad \Leftrightarrow \quad \left[\frac{N_1+N_2}{N_1}\right]^{\frac{1}{\beta}} > \left[1 + \delta\left(\frac{N_2}{N_1\delta} + 1 - \beta\right)^{\frac{1}{\beta}}\right].$$

Also for this comparison, in the range of relevant parameters the $U_1^{SP} > U_1^L$.

In turn when $\frac{N_2}{N_1} > \frac{\beta\delta}{1-\delta}$ the utility comparison is the following:

$$U_1^{SP} > U_1^L \quad \Leftrightarrow \quad 1 - \delta(1-\beta) > (1-\delta)^{1-\beta}.$$

Clearly in this case the differential does not depend on any population variables.

3.5 Costly maintenance: laissez-faire

Since in the baseline case the depreciated stock of publicly provided goods is usable free of charge, the second generation benefits. The incomplete depreciation induces a positive externality. However, most infrastructure cannot continue to be used for generations without maintaining the existing stock, which will incur additional costs. For example, highways or sewage systems may become unusable if investment is not continued (due to potholes or leakages). In the following section I introduce a such a trade-off i.e. costly maintenance and discuss the implications of this additional cost with respect to the no-cost alternative.

Assume that in order to benefit from the level of provision from the previous generation, the second generation has to pay a share in upkeep for the inherited stock of publicly provided goods. Whereas the problem of the first generation remains unchanged, for the second generation the cost of upkeep $q \in (0, 1]$ is an exogenous charge paid on the depreciated provision from the first period. Thus the maintenance charge is a percentage of the stock of the publicly provided good from the previous period and can be considered a running cost on the investment. This charge entails an additional cost in the budget constraint of the second generation:

$$N_2 w = N_2 c_2 + \theta(q\delta G_1 + G_2). \tag{3.22}$$

A higher q reduces the benefit derived from the inheritance. When $q = 0$ the problem reverts back to the baseline case. The utility function of the second generation remains as in equation (3.8).

Since the maintenance cost only affects the second generation the allocations for the first generation remain the same as in the baseline laissez-faire case in Section 3.3 equations (3.6) and (3.7). The second generation public provision is now given by the following expression:

$$G_2^{L,q} = \begin{cases} 0 & \text{if } \frac{N_2}{N_1} \leq \delta\left(\beta + q(1-\beta)\right), \\ \frac{(1-\beta)w}{\theta}\left[N_2 - \delta N_1\left(\beta + q(1-\beta)\right)\right] & \text{if } \frac{N_2}{N_1} > \delta\left(\beta + q(1-\beta)\right). \end{cases} \tag{3.23}$$

The critical level of population loss beyond which $G_2^{L,q} = 0$ is now defined by: $\frac{N_2}{N_1} \leq \delta\left[\beta + q(1-\beta)\right]$. The critical level depends on β and q, and is lower than when no maintenance costs are accounted for ($\beta\delta < \delta(\beta + q(1-\beta))$. Still for this level $\frac{N_2}{N_1} < 1$, such that positive provision takes place even when there is some population loss.

Depending on whether the second generation invests in $G_2^{L,q}$ or relies entirely on the upkeep of the G_1^L also influences the private consumption decision. The private consumption levels are thus also affected by the additional cost:

$$c_2^{L,q} = \begin{cases} w(1 - \frac{N_1}{N_2}q\delta(1-\beta)) & \text{if } \delta q(1-\beta) < \frac{N_2}{N_1} \leq \delta\left(\beta + q(1-\beta)\right), \\ \frac{\beta w}{N_2}\left(N_2 + \delta(1-\beta)N_1(1-q)\right) & \text{if } \frac{N_2}{N_1} > \delta\left(\beta + q(1-\beta)\right). \end{cases} \tag{3.24}$$

In order for the utility of the second generation to be positive, private consumption must take place. When $\frac{N_2}{N_1} > \delta\left(\beta + q(1-\beta)\right)$ also $c_2^{L,q} > 0$ will hold. However, when $\frac{N_2}{N_1} \leq \delta\left(\beta + q(1-\beta)\right)$ there is a lower limit for $c_2^{L,q} > 0$. Only if $\frac{N_2}{N_1} > \delta q(1-\beta)$ can also this condition be fulfilled. If this condition were violated, then the second period consumption would have to be less than zero to meet the cost of upkeep, this however, is infeasible. If population loss is severe and the costs of maintaining infrastructure are high, then "bankruptcy" may arise.

Moreover, the influence of the cost of upkeep on private consumption also depends on whether $G_2^{L,q} \geq 0$. The difference is most conveniently demonstrated using the comparative static effects of changes in N_1 and N_2 on $c_2^{L,q}$. If $\delta q(1-\beta) < \frac{N_2}{N_1} \leq \delta\left(\beta + q(1-\beta)\right)$ the effects are $\frac{\partial c_2^{L,q}}{\partial N_1} < 0$ and $\frac{\partial c_2^{L,q}}{\partial N_2} > 0$. Regardless of the size of the share of upkeep costs, an increase in

N_1 will increase G_1 and hence the costs of upkeep, which in turn implies a reduction in private consumption of the second generation. If N_2 increases then $c_2^{L,q}$ will increase. More users contribute to the upkeep of the inherited stock of publicly provided goods such that more private consumption by each can take place. However, if $\frac{N_2}{N_1} > \delta\left(\beta + q(1-\beta)\right)$ such that $G_2^{L,q} > 0$, the consumption choice is analogous to the baseline case, and the partial effects are $\frac{\partial c_2^{L,q}}{\partial N_1} < 0$ and $\frac{\partial c_2^{L,q}}{\partial N_2} > 0$.

Using the above expressions for $G_2^{L,q}$ and $c_2^{L,q}$ the utility achieved by the second generation is:

$$U_2^{L,q} = \begin{cases} N_2^{-\gamma} w \left(\frac{(1-\beta)\delta N_1}{\theta}\right)^{1-\beta} [N_2 - N_1 q\delta(1-\beta)]^{\beta} & \text{if } \delta q(1-\beta) < \frac{N_2}{N_1} \\ & \quad < \delta\left(q + \beta(1-q)\right), \\ N_2^{-\gamma} w \beta^{\beta} \Big(\frac{1-\beta}{\theta}\Big)^{1-\beta} [N_2 + N_1\delta(1-\beta)(1-q)] & \text{if } \frac{N_2}{N_1} > \delta\left(q + \beta(1-q)\right). \end{cases} \tag{3.25}$$

The following section considers the welfare maximizing outcome of the case with costly upkeep. In a welfare analysis similar to Section 3.4 the laissez-faire solutions are compared to the welfare maximizing solutions with upkeep. The following section is largely analogous to the case when no costs of maintenance are considered and the general case is therefore delegated to the Appendix (see Appendix Section 3.B).

3.6 Costly maintenance: welfare maximization with an intertemporal budget

The results when costs of upkeep are considered show that the lower bound for population loss, to prevent insolvency of the second generation, carries over from the laissez-faire case. In the following the solutions from the above case of laissez-faire generations with upkeep are contrasted with the case in which welfare is maximized.

Again the welfare of the first generation is maximized subject to the aggregate budget constraint modified for costs of upkeep (for the second generation) as well as the constraint on the second generation utility. Thus:

$$\max_{c_1,c_2,G_1,G_2} \quad W^{SP,q} = c_1^{\beta}\left(\frac{G_1}{N_1^{\alpha}}\right)^{1-\beta} \tag{3.26}$$
$$\text{s.t.} \quad w(N_1+N_2) = c_1N_1 + c_2N_2 + \theta((1+q\delta)G_1 + G_2)$$
$$\bar{U}_2 = c_2^{\beta}\left(\frac{\delta G_1 + G_2}{N_2^{\alpha}}\right)^{1-\beta}.$$

To conduct the welfare analysis, the utility maximizing level $U_2^{L,q}$ from equation (3.25) is used to derive specific allocations. For $G_1^{SP,q}$ the following expressions results:

$$G_1^{SP,q} = \begin{cases} \frac{(1-\beta)}{\theta(1+q\delta)} w(N_1+N_2) & \text{if} \quad \delta q(1-\beta) < \frac{N_2}{N_1} < \delta(\beta + q(1-\beta)) \\ w\left(\frac{1-\beta}{\theta}\right) N_1 \left[\frac{1-\delta(1-\beta)(1-q)}{1-\delta(1-q)}\right] & \text{if} \quad \frac{N_2}{N_1} > \delta\left(\beta + q(1-\beta)\right). \end{cases} \tag{3.27}$$

These allocations in the social planner's case with maintenance are similar to those derived in the baseline case. When $\delta q(1-\beta) < \frac{N_2}{N_1} < \delta\left(\beta + q(1-\beta)\right)$ such that $G_2^{SP,q} = 0$, then $G_1^{SP,q} > G_1^L$ will only hold if N_2 is sufficiently large (namely $N_2/N_1 > q\delta$). In contrast to the baseline case, when maintenance costs are accounted for and $N_2/N_1 < q\delta$, the provision for the first generation will be too high in laissez-faire. When $\frac{N_2}{N_1} > \delta\left(\beta + q(1-\beta)\right)$ such that $G_2^{SP,q} > 0$, then $G_1^{SP,q} > G_1^L$. In the lower line of equation (3.27) the term in brackets is greater than one since the numerator is greater than the denominator. Therefore as illustrated in Figure 3.7 the provision of G_1 with maintenance will be higher in the social planner's case than in laissez-faire.

Correspondingly the allocations of the second generation are given by:

$$G_2^{SP,q} = \begin{cases} 0 & \text{if } \delta q(1-\beta) < \frac{N_2}{N_1} \\ & \quad < \delta\left(\beta + q(1-\beta)\right), \\ w\left(\frac{1-\beta}{\theta}\right)\left[N_2 - \frac{N_1\delta(1-(1-\beta)(1+q\delta)(1-q))}{1-\delta(1-q)}\right] & \text{if } \frac{N_2}{N_1} > \delta\left(\beta + q(1-\beta)\right). \end{cases} \tag{3.28}$$

These public goods provision levels with maintenance costs are illustrated in Figure 3.8. When the population relation is given by $\frac{N_2}{N_1} > \delta\left(\beta + q(1-\beta)\right)$ then the provision $G_2^{L,q}$ will be positive. Positive levels of $G_2^{SP,q}$ will be

Figure 3.7: Comparison of the provision levels for the first generation in laissez-faire G_1^L and the social planner's case $G_1^{SP,q}$ with costs of upkeep, plotted against the population relation $\frac{N_2}{N_1}$

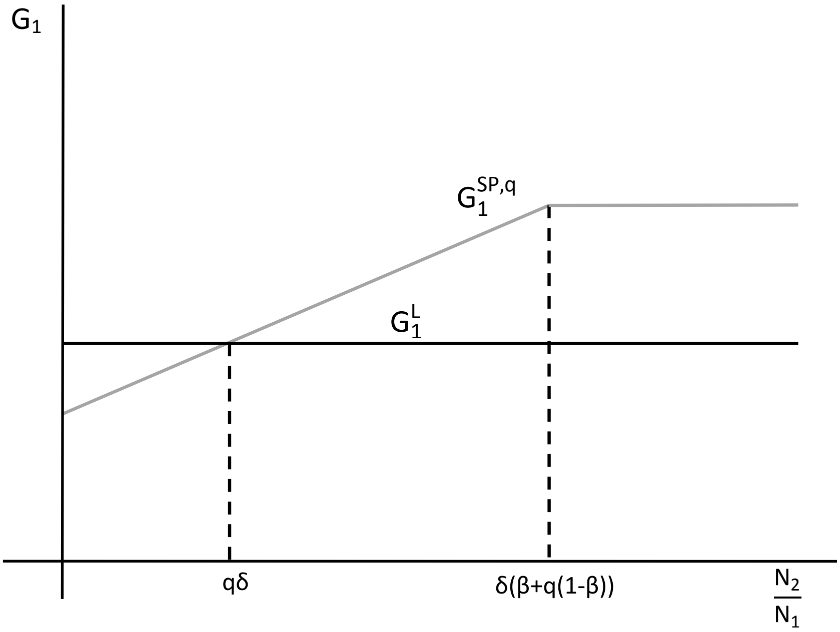
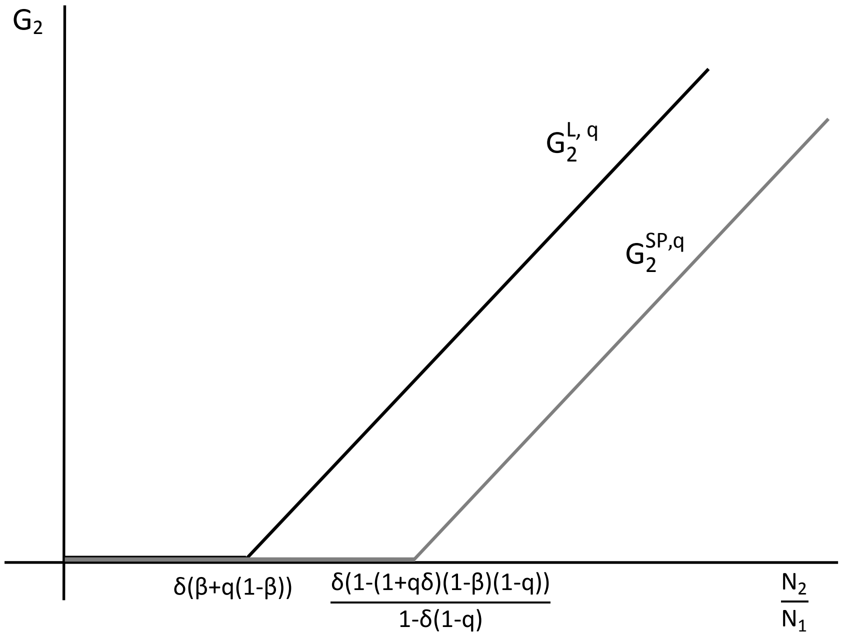


Figure 3.8: Comparison of the provision levels for the second generation in laissez-faire $G_2^{L,q}$ and the social planner's case $G_2^{SP,q}$ with costs of upkeep, plotted against the population relation $\frac{N_2}{N_1}$



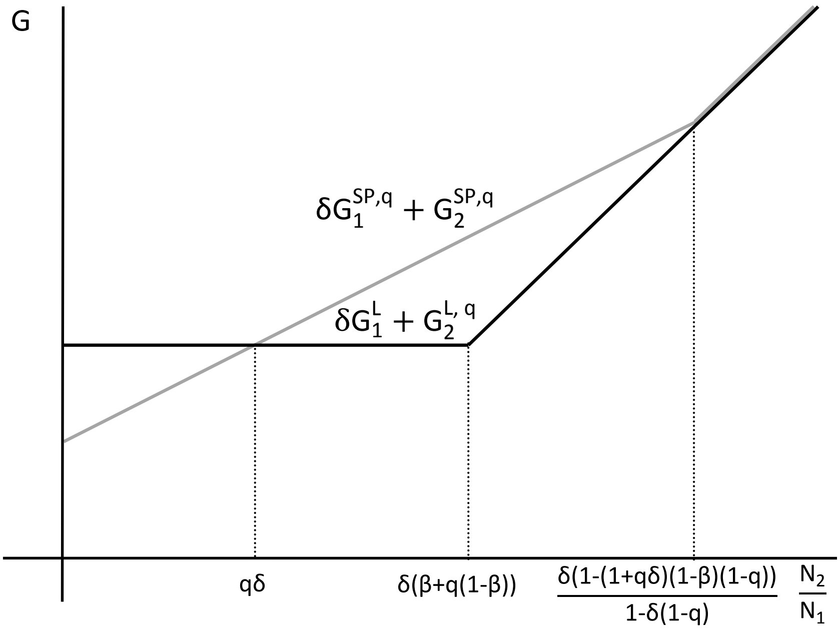
achieved for $\frac{N_2}{N_1} > \frac{\delta(1-(1-\beta)(1+q\delta)(1-q))}{1-\delta(1-q)}$.[9] Thus a positive provision under welfare maximization requires a larger second generation population in comparison to laissez-faire.

With maintenance the aggregate provision level $G = \delta G_1 + G_2$ of the laissez-faire and welfare maximizing outcomes depend on the size of the second generation. This comparison is summarized in:

Proposition 3.3. *i) If $\frac{N_2}{N_1} < q\delta$, then the aggregate provision in laissez-faire is too high compared to the welfare maximizing level.*
ii) If $q\delta < \frac{N_2}{N_1} < \frac{\delta(1-(1-\beta)(1+q\delta)(1-q))}{1-\delta(1-q)}$, then the aggregate provision in laissez-faire is too low compared to the welfare maximizing level.
iii) If $\frac{N_2}{N_1} > \frac{\delta(1-(1-\beta)(1+q\delta)(1-q))}{1-\delta(1-q)}$, then the aggregate provision in laissez-faire is equal to the welfare maximizing level.

9 Whether the threshold $\frac{\delta(1-(1-\beta)(1+q\delta)(1-q))}{1-\delta(1-q)} \gtreqless 1$ is not clear. The value depends on three parameters that can all be between zero and one, therefore parameter constellations yielding $\frac{N_2}{N_1} \gtreqless 1$ are possible.

Figure 3.9: Comparison of the aggregate provision of intergenerational publicly provided goods $G = \delta G_1 + G_2$ with with costs of upkeep in laissez-faire and the social planner's case



Proof: The above proposition results from the aggregate provision of publicly provided goods for the two generations when maintenance is accounted for, in equations (3.27) and (3.28). The comparison of laissez-faire and the social planner's case is illustrated in Figure 3.9.

For $\frac{N_2}{N_1} < \delta q$ the provision under the social planner will now be lower than in the laissez-faire case. This results from the fact that in the welfare maximizing case, the burden of the upkeep costs placed on the second generation is taken into account. Thus for cases of severe population loss the welfare maximizing outcome will allocate fewer resources toward the first generation than would be the case in laissez-faire. For intermediate population relations (relatively stable populations) the publicly provided goods are under-provided in laissez-faire. Finally, for $\frac{N_2}{N_1} > \frac{\delta(1-(1-\beta)(1+q\delta)(1-q))}{1-\delta(1-q)}$ the sum of the provision levels is the same in both the welfare maximizing and laissez-faire cases. Thus when costs of maintenance are accounted for, the public provision in laissez-faire may either be too high, too low or just right.

When the population shrinks the existing infrastructure may be too large, however, it is not clear whether the positive externality from inheritance or the burden from costly upkeep dominates. If $\frac{\delta(1-(1-\beta)(1+q\delta)(1-q))}{1-\delta(1-q)} < \frac{N_2}{N_1} < 1$, then for moderate shrinking the aggregate provision in laissez-faire and the social planner's case is equal. However, if $\frac{\delta(1-(1-\beta)(1+q\delta)(1-q))}{1-\delta(1-q)} > 1$ then population growth is required for the aggregate provision to be equal.

Also the private consumption levels now depend additionally on the cost (q). The specific results are shown in the Appendix 3.C. These consumption levels are in turn used to compute the utility levels of the first generation. Since the threshold values for positive provision for the second generation are again different in laissez-faire and in the welfare maximizing cases, three relevant ranges arise. The utility achieved by the first generation in the social planner's case with costs of upkeep (when $\bar{U}_2 = U_2^{L,q}$) are given by:

$$U_1^{SP,q} = \begin{cases} w\left[N_1+N_2\right]N_1^{-\gamma}\left(\frac{1-\beta}{\theta(1+q\delta)}\right)^{1-\beta}\left[\beta-\left(\frac{N_2-N_1 q\delta(1-\beta)}{N_1(1+q\delta)}\right)\left(\frac{N_1(1+q\delta)}{N_1+N_2}\right)^{\frac{1}{\beta}}\right]^{\beta} \\ \qquad \text{if} \quad \delta q(1-\beta) < \frac{N_2}{N_1} < \delta\left(q+\beta(1-q)\right), \\ w\left[N_1+N_2\right]N_1^{-\gamma}\left(\frac{1-\beta}{\theta(1+q\delta)}\right)^{1-\beta}\left[\beta-\left(\frac{\delta\beta}{1+q\delta}\right)\left(\frac{(1+q\delta)(N_2+N_1\delta(1-\beta)(1-q))}{\delta(N_1+N_2)}\right)^{\frac{1}{\beta}}\right]^{\beta} \\ \qquad \text{if} \quad \delta\left(q+\beta(1-q)\right) < \frac{N_2}{N_1} \leq \frac{\delta(1-(1-\beta)(1+q\delta)(1-q))}{1-\delta(1-q)}, \\ N_1^{1-\gamma}w\beta^{\beta}\left(\frac{(1-\beta)}{\theta(1-\delta(1-q))}\right)^{1-\beta}\left[1-\delta(1-\beta)(1-q)\right] \\ \qquad \text{if} \quad \frac{N_2}{N_1} > \frac{\delta(1-(1-\beta)(1+q\delta)(1-q))}{1-\delta(1-q)}. \end{cases} \tag{3.29}$$

As in the baseline case a comparison of the utility levels of the first generation in the social planner's and laissez-faire settings follows. When the size of the population of the second generation is such that: $\delta q(1-\beta) < \frac{N_2}{N_1} < \delta\left[\beta + q(1-\beta)\right]$, the utility comparison of the first generation is the following:

$$U_1^{SP,q} \geq U_1^L \quad \Leftrightarrow \quad \beta\left[\left(\frac{N_1+N_2}{N_1}\right)^{\frac{1}{\beta}}(1+q\delta)^{\frac{\beta-1}{\beta}} - 1\right] > \frac{N_2}{N_1} - q\delta(1-\beta).$$

For some parameter constellations of (β, δ and q) equality is possible, otherwise $U_1^{SP,q} > U_1^L$. The difference in utility is rising in the second generation population.

When the population relation lies in the intermediate range, the utility comparison is the following:

$$U_1^{SP,q} > U_1^L \Leftrightarrow$$
$$\left(\frac{N_1+N_2}{N_1}\right)^{\frac{1}{\beta}} > (1+q\delta)^{\frac{1-\beta}{\beta}}\left[1+\delta\left(\frac{N_2}{N_1\delta}+(1-\beta)(1-q)\right)^{\frac{1}{\beta}}\right].$$

Similarly when $\frac{N_2}{N_1} > \frac{\delta(1-(1-\beta)(1+q\delta)(1-q))}{1-\delta(1-q)}$ the utility comparison gives:

$$U_1^{SP,q} > U_1^L \quad \Leftrightarrow \quad \left[\frac{1-\delta(1-\beta)(1-q)}{1-\delta(1-\beta)}\right] > (1-\delta(1-\beta))^{\beta}.$$

Again the difference is independent of the size of the population when publicly provided goods are provided in both periods.

3.7 Conclusion

In this chapter a simple model on the provision of intergenerational publicly provided goods was developed. The focus laid on the effect of local population loss on the provision. In the laissez-faire case, the assumed intergenerational link is simply the inherited stock of publicly provided goods. In the welfare maximizing case, the social planner additionally allocates the aggregated budgets of the two generations. Moreover, the outcomes of the baseline case were contrasted with the results when costs of upkeep accrue to the ensuing generation.

The choices of private consumption and public goods provision are different when the population shrinks in comparison to cases of population growth or stability. In a laissez-faire setting, if a jurisdiction is experiencing population growth, public goods will always be provided in both periods. If, however, the community experiences population loss, it may be optimal from the view of an ensuing generation not to provide any additional public goods. Conversely under welfare maximization the resources are shifted between provision for each generation depending on their size. Thus in comparison to welfare maximization, in laissez-faire the provision of intergenerational public goods will be inefficiently low from the view of the next generation when no costs accrue. However, when maintenance is expensive successive generations may prefer less provision by past generations.

When provision toward the second generation does take place, i.e. the population is growing or stable then the welfare differential between laissez-faire and the social planner remains constant with respect to the population size change. However, when population loss is so severe that no publicly provided goods are provided for the second generation the loss in welfare increases with further population loss. Hence, in times of severe population loss intergenerational planning of long term investments is paramount.

Since a welfare improvement may result when the resources of both generations are allocated at the beginning of the first period, and the ensuing generation benefits from the previous investment, the setup can be used to justify debt. Today debt is raised to pay for infrastructure that will outlast the current generation. Hence the future generations contribute by paying back the debt incurred to build up the stock from which they derive utility. However, it was also shown that in situations with high population loss, the ensuing generations may become too small to effectively benefit from existing stock of public goods. In such cases smaller investments in previous periods would suffice.

The demographic risk due to a smaller future population may thus also contain a fiscal risk. To prevent mis-investments in times when future population loss is likely, a comprehensive evaluation of demographic risk could be conducted. Local governments could commit to evaluate (infrastructure) investment decisions, with respect to their demographic sustainability (aging and shrinking) before approval. In light of the projected population developments in many developed countries with extensive infrastructures, dealing with the consequences of population loss will become an important policy issue. More careful weighing of current and future needs for investment, accompanied by a more detailed consideration of future population developments particularly at the local level, should become more common.

The model developed here is admittedly simple. Considering only two generations may mask longer term effects. Therefore, similar issues could be considered with a model in multi-period setting. Furthermore, a more sophisticated cost function in lieu of the exogenous cost share used here could allow more detailed analysis of the incentives to shift resources between current and future consumption.

Appendix

Appendix 3.A Welfare maximization with an exogenous second generation utility level

3.A.1 Provision levels of the publicly provided goods

In the following the welfare maximization problem from Section 3.4 equation (3.16), is solved for a general exogenously given $\bar{U_2}$. A non-negativity restriction defines the range of $\bar{U_2}$ for which feasible allocations are achieved (as shown below). The provision level G_1^{SP} is given by the below expressions depending on whether $G_2^{SP} \geq 0$, the superscript SP indicates the case of the social planner:

$$G_1^{SP} = \begin{cases} \frac{(1-\beta)}{\theta} w(N_1 + N_2) & \text{if} \quad \bar{U_2} \leq \delta z, \\ \frac{1}{1-\delta} \left(\frac{1-\beta}{\theta\beta} \right)^{\beta} N_2^{\gamma} \left[\phi w(N_1 + N_2) N_2^{-\gamma} - \bar{U_2} \right] & \text{if} \quad \delta z < \bar{U_2} < z. \end{cases} \tag{3.30}$$

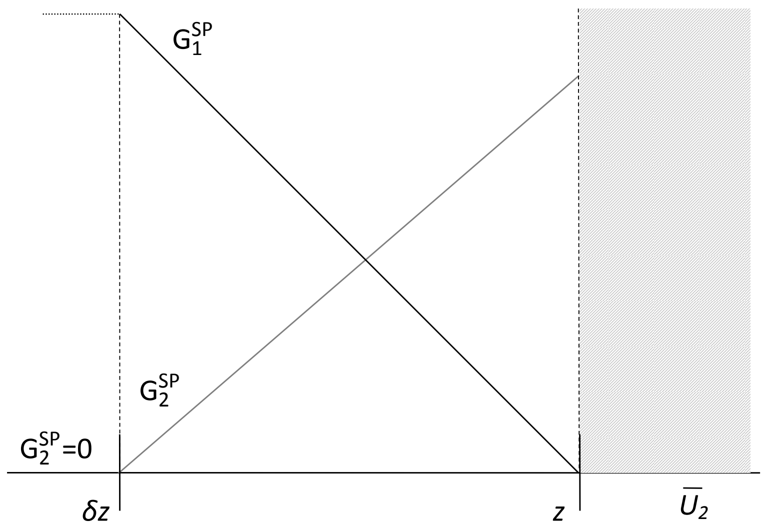
For clarity of the exposition I consolidate the parameters by defining $\phi \equiv \left(\frac{1-\beta}{\theta}\right)^{1-\beta} \beta^{\beta}$ and again the exponent $\gamma \equiv \beta(1-\alpha) + \alpha$. The provision of G_1^{SP} may not be zero, because otherwise the whole generational utility would be zero. However, due to the intergenerational character of the provision in the second period, $G_2^{SP} = 0$ may be the solution, if the inherited G_1^{SP} is sufficient to satisfy $\bar{U_2}$. Thus when $G_2^{SP} = 0$, G_1^{SP} is given by the top line in equation (3.30). Solving for G_2^{SP} gives:

$$G_2^{SP} = \begin{cases} 0 & \text{if} \quad \bar{U_2} \leq \delta z, \\ \frac{1}{1-\delta} \left(\frac{1-\beta}{\theta\beta} \right)^{\beta} N_2^{\gamma} \left[\bar{U_2} - \delta\phi w(N_1 + N_2) N_2^{-\gamma} \right] & \text{if} \quad \delta z < \bar{U_2} < z. \end{cases} \tag{3.31}$$

Together the above provision levels imply that $G_1^{SP}, G_2^{SP} > 0$ only if:

$$\delta\phi w(N_1 + N_2) N_2^{-\gamma} \; < \; \bar{U_2} \; < \; \phi w(N_1 + N_2) N_2^{-\gamma}. \tag{3.32}$$

Figure 3.10: Provision levels G_1^{SP} and G_2^{SP} for the first and second generations in the feasible range $\delta z < \bar{U}_2 < z$



where $\phi w(N_1 + N_2)N_2^{-\gamma} \equiv z$. At the extremes of the range $\delta z < \bar{U}_2 < z$ one of the provision levels will be equal to zero, at $\bar{U}_2 = \delta z$, $G_1^{SP} > 0$ and $G_2^{SP} = 0$, and at $\bar{U}_2 = z$, $G_1^{SP} = 0$ and $G_2^{SP} > 0$. If $\bar{U}_2 > z$ it implies that the exogenously chosen level of reservation utility for the second generation is so high that not enough resources remain to provide any G_1^{SP}. Beyond $\bar{U}_2 = z$ when $G_1^{SP} = 0$ the utility of the first generation will be zero, and therefore feasible allocations only arise when $\bar{U}_2 \leq z$.

Figure 3.10 provides a stylized illustration of the provision levels. The horizontal axis depicts the level of $\bar{U}_2$. This line is intersected by two dashed vertical lines that mark the range of values (i.e. $\delta z < \bar{U}_2 < z$) for which both $G_1^{SP} > 0$ and $G_2^{SP} > 0$. The two intersecting lines represent the provision levels, where G_1^{SP} is given by the downward sloping black line that ends at z and G_2^{SP} by the upward sloping gray line. Beyond the relevant range, either G_1^{SP} or G_2^{SP} would equal zero. The shaded area to the right of z shows that $G_1^{SP} = 0$ is not feasible. In contrast, to the left of δz, $G_2^{SP} = 0$ but the level of provision for G_1^{SP} is positive and feasible.

Moreover, the amount invested in G_1^{SP} depends on the rate of depreciation

δ. When $0 < \delta < 1$ the provision levels G_1^{SP} and G_2^{SP} of the intergenerational publicly provided good are given by the bottom expressions in (3.30) and (3.31). When $\delta \to 1$, $G_2^{SP} \to 0$ as is clear from (3.31), and correspondingly G_1^{SP} is given by the upper line in (3.30). If $\delta = 0$, the publicly provided good is not intergenerational, and the provision for the second generation is equal to $G_2^{SP} = \left(\frac{1-\beta}{\theta\beta}\right)^{\beta} N_2^{\gamma}\bar{U}_2$. G_1^{SP} is increasing in δ and consequently G_2^{SP} is decreasing in δ. For low rates of depreciation less will be inherited and there will be more investment in the publicly provided good for the second generation.

3.A.2 Private consumption

I also derive the levels of private consumption in the welfare maximizing case. For the first generation the following private consumption levels are found:

$$c_1^{SP} = \begin{cases} w\left(1+\frac{N_2}{N_1}\right)\left[\beta - N_2\left(\frac{\delta(1-\beta)}{\theta}\right)^{\frac{\beta-1}{\beta}}\left(\frac{\bar{U}_2 N_2^{\alpha(1-\beta)}}{w(N_1+N_2)}\right)^{\frac{1}{\beta}}\right] & \text{if } \bar{U}_2 \leq \delta z, \\ \frac{1}{N_1}\left[\beta w(N_1+N_2) - \bar{U}_2\left(\frac{1-\beta}{\theta\beta}\right)^{\beta-1} N_2^{\gamma}\right] & \text{if } \delta z < \bar{U}_2 < z. \end{cases} \tag{3.33}$$

The effects of changes in N_1 and N_2 on c_1^{SP} depend on which range is considered, but in either case the effects are ambiguous.

The corresponding private consumption of the second generation is given by:

$$c_2^{SP} = \begin{cases} \left[\bar{U}_2 N_2^{\alpha(1-\beta)}\left(\delta\left(\frac{1-\beta}{\theta}\right) w(N_1+N_2)\right)^{\beta-1}\right]^{\frac{1}{\beta}} & \text{if } \bar{U}_2 \leq \delta z, \\ \left(\frac{1-\beta}{\theta\beta}\right)^{\beta-1}\left[\bar{U}_2 N_2^{\gamma-1}\right] & \text{if } \delta z < \bar{U}_2 < z. \end{cases} \tag{3.34}$$

3.A.3 First generation utility

Given the second constraint of the problem, the second period utility is fixed at some value $\bar{U}_2$. However, the first period utility depends on the population and the parameters of the model as well as this $\bar{U_2}$. By inserting the derived provision and consumption levels into the utility function of the first generation, the following expressions result:

$$U_1^{SP} = \begin{cases} N_1^{-\gamma} w(N_1+N_2)\left(\frac{1-\beta}{\theta}\right)^{1-\beta}\left[\beta - N_2\left(\frac{\delta(1-\beta)}{\theta}\right)^{\frac{\beta-1}{\beta}}\left(\frac{\bar{U_2}N_2^{\alpha(1-\beta)}}{w(N_1+N_2)}\right)^{\frac{1}{\beta}}\right]^{\beta} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{if} \quad \bar{U_2} \leq \delta z, \\ \left(\frac{N_2}{N_1}\right)^{\gamma}(1-\delta)^{\beta-1}\left[\phi N_2^{-\gamma} w(N_1+N_2) - \bar{U_2}\right] \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{if} \quad \delta z < \bar{U_2} < z. \end{cases} \tag{3.35}$$

In all of the above expressions $\bar{U_2} = U_2^L$ is substituted in the relevant range to give the expressions found in Section 3.4.

Appendix 3.B Welfare maximization and analysis with costly maintenance

The welfare of the first generation is maximized subject to the budget constraint modified for costs of upkeep and the constraint on the second generation utility which also accounts for the costs q. Thus:

$$\begin{aligned} \max_{c_1,c_2,G_1,G_2} \quad & W^{SP,q} = c_1^{\beta}\left(\frac{G_1}{N_1^{\alpha}}\right)^{1-\beta} \qquad (3.36) \\ \text{s.t.} \quad & w(N_1+N_2) = c_1N_1 + c_2N_2 + \theta(G_1 + q\delta G_1 + G_2) \\ & \bar{U_2} = c_2^{\beta}\left(\frac{\delta G_1 + G_2}{N_2^{\alpha}}\right)^{1-\beta}. \end{aligned}$$

The provision of publicly provided good $G_1^{SP,q}$ is now given by:

$$G_1^{SP,q} = \begin{cases} \frac{(1-\beta)}{\theta(1+q\delta)} w(N_1+N_2) & \text{if} \quad \bar{U_2} < \frac{\delta}{1+q\delta} z \\ \frac{1}{1-\delta(1-q)}\left(\frac{1-\beta}{\theta\beta}\right)^{\beta} N_2^{\gamma}\left[\phi w(N_1+N_2)N_2^{-\gamma} - \bar{U_2}\right] & \text{if} \quad \frac{\delta}{1+q\delta} z < \bar{U_2} < z. \end{cases} \tag{3.37}$$

Save for the expression for the depreciation the provision levels are identical.

When $\frac{\delta}{1+q\delta}z < \bar{U}_2 < z$, the denominator of the first fraction to the left, reflects the additional term from the costly upkeep. The cost of upkeep will result in a lower provision of G_1 than in the case where potential costs are not taken into account. Furthermore the additional cost factor is also involved in the provision of $G_1^{SP,q}$ even when $\bar{U}_2 < \frac{\delta}{1+q\delta}z$. Due to the additional cost the total budget available is lower.

Similarly the second generation provision levels are now given by:

$$G_2^{SP,q} = \begin{cases} 0 & \text{if} \quad \bar{U}_2 < \frac{\delta}{1+q\delta}z, \\ \frac{1}{1-\delta(1-q)} \left(\frac{1-\beta}{\theta\beta}\right)^{\beta} N_2^{\gamma} \left[\bar{U}_2(1+q\delta) \right. & \\ \qquad \left. -\delta\phi w(N_1+N_2)N_2^{-\gamma}\right] & \text{if} \quad \frac{\delta}{1+q\delta}z < \bar{U}_2 < z. \end{cases} \tag{3.38}$$

Therefore the range in which both G_1^{SP} and G_2^{SP} are positive is now:

$$\frac{\delta}{1+q\delta}\phi w(N_1+N_2)N_2^{-\gamma} < \bar{U}_2 < \phi w(N_1+N_2)N_2^{-\gamma}.$$

The first generation consumes according to:

$$c_1^{SP,q} = \begin{cases} \frac{w(N_1+N_2)}{N_1}\left[\beta - N_2\left(\frac{\delta(1-\beta)}{\theta(1+q\delta)}\right)^{\frac{\beta-1}{\beta}}\left(\frac{\bar{U}_2 N_2^{\alpha(1-\beta)}}{w(N_1+N_2)}\right)^{\frac{1}{\beta}}\right] & \text{if } \bar{U}_2 < \frac{\delta}{1+q\delta}z, \\ \frac{1}{N_1}\left[\beta w(N_1+N_2) - \bar{U}_2\left(\frac{1-\beta}{\theta\beta}\right)^{\beta-1} N_2^{\gamma}\right] & \text{if } \frac{\delta}{1+q\delta}z < \bar{U}_2 < z, \end{cases} \tag{3.39}$$

analogously for the second generation:

$$c_2^{SP,q} = \begin{cases} \left[\bar{U}_2 N_2^{\alpha(1-\beta)}\left(\frac{\delta(1-\beta)}{\theta(1+q\delta)}w(N_1+N_2)\right)^{\beta-1}\right]^{\frac{1}{\beta}} & \text{if} \quad \bar{U}_2 < \frac{\delta}{1+q\delta}z, \\ \left(\frac{1-\beta}{\theta\beta}\right)^{\beta-1}\left[\bar{U}_2 N_2^{\gamma-1}\right] & \text{if} \quad \frac{\delta}{1+q\delta}z < \bar{U}_2 < z. \end{cases} \tag{3.40}$$

Hence the expressions for $U_1^{SP,q}$ are given by:

$$U_1^{SP,q} = \begin{cases} w(N_1+N_2)N_1^{\beta(\alpha-1)-\alpha}\left(\frac{1-\beta}{\theta(1+q\delta)}\right)^{1-\beta} \\ \quad \left\{\beta - N_2\left[\frac{\delta(1-\beta)}{\theta(1+q\delta)}\right]^{\frac{\beta-1}{\beta}}\left[\frac{\bar{U}_2 N_2^{\alpha(1-\beta)}}{w(N_1+N_2)}\right]^{\frac{1}{\beta}}\right\}^{\beta} & \text{if} \quad \bar{U}_2 < \frac{\delta}{1+q\delta}z, \\ N_1^{-\gamma}(1-\delta(1-q))^{\beta-1} \\ \quad \left[\left(\frac{1-\beta}{\theta}\right)^{1-\beta}\beta^{\beta} w\left(N_1+N_2\right) - \bar{U}_2 N_2^{\gamma}\right] & \text{if} \quad \frac{\delta}{1+q\delta}z < \bar{U}_2 < z. \end{cases} \tag{3.41}$$

Appendix 3.C Private consumption levels with costly maintenance when the second generation utility level is equal to the laissez-faire outcome

The private consumption levels of the two generations are given by the following two expressions when the utility of the second generation achieved laissez-faire is equated to the exogenous utility level $\bar{U}_2$:

$$c_1^{SP,q} = \begin{cases} w\left(\frac{N_1+N_2}{N_1}\right)\left\{\beta - \left[\frac{N_2-N_1 q\delta(1-\beta)}{N_1(1+q\delta)}\right]\left[\frac{N_1(1+q\delta)}{N_1+N_2}\right]^{\frac{1}{\beta}}\right\} \\ \qquad \text{if} \quad \delta q(1-\beta) < \frac{N_2}{N_1} \leq \delta\left(q+\beta(1-q)\right) \\ \beta w\left(\frac{N_1+N_2}{N_1}\right)\left\{1 - \frac{\delta}{1+q\delta}\left[\frac{1+q\delta}{\delta}\left(\frac{N_2+N_1\delta(1-\beta)(1-q)}{N_1+N_2}\right)\right]^{\frac{1}{\beta}}\right\} \\ \qquad \text{if} \quad \delta\left(q+\beta(1-q)\right) < \frac{N_2}{N_1} \leq \frac{\delta(1-(1-\beta)(1+q\delta)(1-q))}{1-\delta(1-q)} \\ \beta w\left[1-\delta(1-\beta)(1-q)\right] \quad \text{if} \quad \frac{N_2}{N_1} > \frac{\delta(1-(1-\beta)(1+q\delta)(1-q))}{1-\delta(1-q)}. \end{cases} \tag{3.42}$$

$$c_2^{SP,q} = \begin{cases} \frac{w}{N_2}\left(\frac{\delta(1-\beta)}{\theta}\right)\left[\frac{N_1+N_2}{(1+q\delta)}\right]^{\frac{\beta-1}{\beta}}(N_2 - N_1 q\delta(1-\beta)) \\ \qquad\qquad \text{if} \quad \delta q(1-\beta) < \frac{N_2}{N_1} \leq \delta\left(q+\beta(1-q)\right), \\ \beta w \frac{\delta}{1+q\delta}\left(\frac{N_1+N_2}{N_2}\right)\left[\left(\frac{\delta}{1+q\delta}\right)\left(\frac{N_2+N_1\delta(1-\beta)(1-q)}{N_1+N_2}\right)\right]^{\frac{1}{\beta}}(N_2 - N_1 q\delta(1-\beta)) \\ \qquad\qquad \text{if} \quad \delta\left(q+\beta(1-q)\right) < \frac{N_2}{N_1} \leq \frac{\delta(1-(1-\beta)(1+q\delta)(1-q))}{1-\delta(1-q)}, \\ w(1-\frac{N_1}{N_2}q\delta(1-\beta)) \qquad \text{if} \quad \frac{N_2}{N_1} > \frac{\delta(1-(1-\beta)(1+q\delta)(1-q))}{1-\delta(1-q)}. \end{cases} \tag{3.43}$$

Chapter 4

Determinants of efficiency in child care provision

4.1 Introduction

During the past three decades most developed countries have had sustained below-replacement fertility. One reason for the low fertility rates is the difficulty to reconcile careers and family life. Governments can implement family policy to encourage both female labor market participation and fertility. One such instrument is public child care provision. In addition to the positive effect on fertility (Hank *et al.*, 2003) and increased labor market participation of women (Minagawa and Upmann, 2006), child care also has a positive influence on the children's future labor market performance (Heckman and Masterov, 2007).[1]

Whereas family policy is a national issue, the provision of child care is usually delegated to the local level. Aging and shrinking populations present new challenges to local policy makers (Geys *et al.*, 2008). Nonetheless, municipalities are legally required to provide certain services and are thus limited

[1] This Chapter is based on the paper "Determinants of efficiency in child care provision", which appeared as *ifo Working Paper Number 83* in March 2010 and has subsequently been published in *Finanzarchiv/Public Finance Analysis (FA)*. The paper was co-authored by Christian Thater (see Montén and Thater (2010) and Montén and Thater (2011)).

in their maneuverability to the allocation of a set of resources between mandated activities. Given the heterogeneity in the size and socio-demographic structure of municipalities, differences in expenditure decisions are to be expected (Oates, 1972). A municipality that is more efficient in the allocation of its resources may thus be able to provide more child care services with fewer resources than a comparable less efficient municipality.[2]

In the coming years, the federal government of Germany hopes to significantly expand the availability of child care. The aim is to facilitate the re-entry of young mothers into the labor force and to support the reconciliation of work and family life. By the end of 2013, the parents of all children between one and six years will have a legal claim to public child care (KiföG, 2008). The municipalities are in charge of the implementation of this expansion and will therefore be faced with increasing expenditures. One possibility to increase the number of places offered without substantially increasing the expenditures is to raise the efficiency in the public provision of child care. In this chapter, we use a unique data set on child care expenditures in the eastern German state of Saxony to assess the efficiency of service provision.

In spite of the obvious importance of the child care sector, only a few studies have attempted to analyze its efficiency. To our knowledge, the only study that analyzes the efficiency of child care centers is Bjurek *et al.* (1992). They investigate the efficiency of expenditures of child care facilities in the city region of Gothenburg in Sweden. Using data on the facility level, they find that the same level of input could produce 10 to 15% more output and that centers in more affluent areas and centers with a more experienced

[2] In recent years, the studies analyzing local government efficiency have expanded. Some authors have considered the overall efficiency of the local public sector, such as De Borger and Kerstens (1996), Geys (2006) and Geys and Moesen (2009), the Belgian local government, Kalb *et al.* (2012) German municipalities, Sampaio de Sousa and Stosic (2005) Brazilian municipalities, Worthington (2000) the Australian local government, Afonso and Fernandes (2008) and the Portuguese local government. The last mentioned and Kalb *et al.* (2012) also contain a more comprehensive surveys of relevant literature. Others focus on the provision of specific services, such as police protection (Drake and Simper, 2003), public libraries (Hemmeter, 2006; De Witte and Geys, 2011), street-lighting (Lorenzo and Sánchez, 2007), county roads (Kalb, 2009) or public schools (Millimet and Collier, 2008).

director are more efficient.[3] Because we consider municipalities instead of the individual facilities, the factors that influence efficiency are not directly comparable to those considered here. Our focus on municipalities is in line with Millimet and Collier (2008), who stress the importance of the school district as the financier of education as the correct level of analysis as opposed to the schools themselves.

In a two-stage analysis, we evaluate the efficiency of municipalities in the provision of child care services. First, we employ a Data Envelopment Analysis (DEA) to compute the efficiency scores, and then, in the second stage, we implement the bootstrap procedure proposed by Simar and Wilson (2007) to investigate the determinants of efficiency in a truncated regression. We find significant efficiency reserves, but the differences between the municipalities are large. In the median municipality, a given level of provision could be achieved with 22-25% lower inputs. Because quality differences may influence the efficiency of service provision, we test whether this is the case in our sample. Using data on children's school readiness, we do not find evidence for differences in the quality of care between the municipalities. Second, we use economic and socio-demographic variables to explain the differences in efficiency. In particular, the lack of professionalism of the mayor and a larger share of the elderly population in a given municipality have a negative impact on the efficiency.

This chapter is structured as follows: Section 4.2 presents both the methods of efficiency analysis as well as the second stage regression. In Section 4.3, we introduce the data and discuss the results of the efficiency analysis. The variables used in the second stage and results of the regression are discussed in Section 4.4. The final Section 4.5 concludes with a discussion and policy implications.

[3] The results they obtain in the second stage efficiency determination analysis are to be viewed with caution. Methodological developments in recent years have shown that the adopted Tobit specification and the serial correlation of the efficiency scores may induce biased results (Simar and Wilson, 2007).

4.2 Methods

4.2.1 Efficiency analysis

Generally efficiency analysis is concerned with the measurement of an organization's ability to use its inputs to produce outputs. In the efficiency analysis literature, two main methods have been employed to measure an organization's ability to use its inputs to produce outputs: the non-parametric Data Envelopment Analysis (DEA) (Farrell, 1957; Charnes *et al.*, 1978; Banker *et al.*, 1984) and the parametric Stochastic Frontier Analysis (SFA) (Aigner *et al.*, 1977; Meeusen and van Den Broeck, 1977; Battese and Corra, 1977). The main benefit of using a stochastic approach is that a deviation from the frontier can be identified as either statistical noise or technical inefficiency. In DEA, by contrast, all deviation is considered inefficiency. However, an advantage of the non-parametric approach lies in the flexibility to model multiple inputs and outputs when prices are not available. In this respect, the SFA is less flexible, as a specific functional form must be chosen to estimate the proposed cost or production function.[4] As the form of the production function is not obvious when considering public units, in this case, it is preferable to employ non-parametric methods where assumptions need only to be made with regards to the properties of the points in the production set (disposability, proportionality or convexity) (Pestieau, 2009). For these reasons, the non-parametric method has been prevalent among studies considering the public sector. In the current context, the efficiency will be evaluated with a DEA.

Due to the well-known difficulties in defining inputs and outputs for public goods, the method of efficiency analysis is not beyond critique (Pestieau, 2009). The risk of mis-specifying the efficient frontier can be reduced by two means. Firstly, the more narrowly the public service is defined, the more closely appropriate inputs and outputs can be matched to the service. Secondly, testing different combinations of inputs and outputs increases the

[4] Usually either a Cobb-Douglas or the more flexible trans-logarithmic specification is chosen (Christensen *et al.*, 1973).

reliability of the specified efficient frontier. Hence, we evaluate the efficiency in the provision of the relatively narrowly defined category child care and test several different model specifications.

4.2.2 Data envelopment analysis (DEA)

In the application of a DEA, some critical assumptions regarding the production process need to be made. The process may either be input or output oriented. Input orientation implies that a frontier is generated based on those observations that use the lowest mix of inputs to produce their outputs. Alternatively, in the output orientation, those observations that achieve the highest mix of outputs given the level of inputs are evaluated. The choice of orientation depends on the objective or the dimension in which the policy-maker is believed to have more discretion (Worthington and Dollery, 2000; Fried *et al.*, 2008). Furthermore, an assumption on the convexity of the production set determines the returns to scale. We test assumptions on both constant returns to scale (CRS) (Charnes *et al.*, 1978) and variable returns to scale (VRS) (Banker *et al.*, 1984) (accounting for both increasing and decreasing returns to scale). Figure 4.1 in Appendix 4.B illustrates the CRS and VRS frontiers for the production of one output with one input.[5]

The efficiency of the units in the sample is assessed in two steps. In the first step, a frontier is generated based on those observations that use the lowest mix of inputs to produce their outputs (input orientation).[6] In the second step, each observation is compared to the piece-wise linear surface of the efficient observations derived in the first step, and then each is assigned an efficiency score. By solving a distinct linear program for each unit, the efficiency of each is maximized by finding the best possible weights of inputs and outputs. The procedure is constrained by the condition that when all units receive the weights that maximize their respective efficiency, none may receive an efficiency score greater than 1. Generally, all weights are non-

[5] In section 4.3.4, the results from both input and output orientations and the results of the CRS and VRS specifications are discussed.

[6] Alternatively, those observations that achieve the highest mix of outputs given the level of inputs (output orientation) could be evaluated.

negative, and no set of other weights will render a higher efficiency. Thus, the frontier is the set of efficient units "enveloping" those that are not as efficient.

The inefficiency of a unit (in this case a municipality) is thus the distance from the efficient surface, or its input-output ratio, in comparison to the units (municipalities) that lie on the surface. Inefficiency (in input orientation) is how much a unit (municipality) could reduce inputs while still achieving the current output level.[7] The problem below formally describes the envelopment form of the input oriented variable returns to scale model according to Banker *et al.* (1984):

$$\begin{aligned} \min_{\theta,\lambda} \quad & \theta, \\ \text{s.t.} \quad & -y_i + Y\lambda \geq 0, \\ & \theta x_i - X\lambda \geq 0 \\ & N1'\lambda = 1 \\ & \lambda \geq 0. \end{aligned} \tag{4.1}$$

θ is a scalar and is the efficiency score of the i-th unit. $\theta \leq 1$, where a value of 1 indicates a technically efficient unit that lies on the frontier. A set of N municipalities uses M inputs and generates S outputs. Then for the i-th municipality, the known inputs and outputs are represented by the vectors x_i and y_i, respectively. For all N units, the input matrix X has the dimensions $(M \times N)$, and the output matrix Y is represented by an $(S \times N)$ matrix. λ is a $(N \times 1)$ vector of constraints, and $N1$ is an $(N \times 1)$ vector of ones.[8] Thus, a convex hull that envelops the data points is constructed as described above.

The information contained in the computed efficiency scores shed only

7 In the output orientation, this refers to how much it could increase its output while not employing more inputs.

8 If CRS are assumed, the model is computed without the convexity constraint $N1'\lambda = 1$ according to Charnes *et al.* (1978). The CRS specification is only valid when all efficient units are operating on this ray; when this is not the case, a variable returns to scale specification is more appropriate.

limited light on the sources of inefficiency, as the decision-making units (DMUs) are traditionally treated as a black box turning inputs into outputs at different levels of efficiency (Fried *et al.*, 2008). Recently, however, the development of methods for conducting a statistically sound explanatory analysis has contributed to attempts at identifying the sources of (in)efficiency (e.g. Simar and Wilson (2007)). In such a two-stage approach, factors that influence the efficiency (environmental variables) beyond the direct production process are regressed on the efficiency scores from the first stage to describe the conditions that are more and less suitable for efficient outcomes. The set of explanatory variables used here is discussed in Section 4.4.

4.2.3 Second stage regression analysis: Estimation procedure

The naive OLS regression or Tobit regression of DEA efficiency scores in a two-stage approach is to be viewed critically (Fried *et al.*, 2008; Simar and Wilson, 2007). However, once the serial correlation of the efficiency scores is accounted for by an appropriate bootstrapping procedure, the use of a second stage regression becomes legitimate. The basis of such a two-stage approach is the assumption that the DMUs face certain environmental variables (z) that constrain their choices of inputs (x) and outputs (y). In other words, the variables in z influence the mean and the variance of the inefficiency process but do not influence the production process itself.[9] To estimate the model, we specify a truncated regression. Formally, the observations stem from the set $S_n = (x_i, y_i, z_i)_{i=1}^n$, where x_i and y_i are the inputs and outputs, used in the i-th unit (municipality) to derive the efficiency (θ_o) of each of the n observations in the previous section. Additionally, the observations are characterized by certain environmental variables contained in the vector z_i.

The problems that arise when conducting a naive second stage regression include: slow convergence of the estimated parameters towards the true values when more than one input and output are included, and serial cor-

[9] For example, a municipality's source of revenues, whether from taxes or transfers, does not influence the production of child care but may influence the incentives to use the resources efficiently.

relation of unknown form between the efficiency scores and the explanatory variables. Moreover, by construction a change in one efficiency score can change the whole frontier. In other words the efficiency of all units are dependent on all other units. Since several efficiency scores equal one, it may suggest a censoring at the probability mass of 1. Some authors therefore use a Tobit specification in the second stage regression (e.g., Bjurek *et al.*, 1992; De Borger and Kerstens, 1996; Kirjavainen and Loikkanen, 1998; Worthington and Dollery, 2001). However, since by construction an efficiency score cannot exceed 1 the dependent variable is in fact truncated and not censored. Therefore a truncated regression is more appropriate, and has also been shown to perform better in simulations (Simar and Wilson, 2007).

In the input orientation the dependent variable (the inverse of the efficiency score) is the obtained by the input distance function and hence $\theta = (1, \infty)$ (Shephard, 1970). The efficiencies (θ_i) are in this case computed within the production possibilities (P) according to:

$$\theta_i = \theta(x_i, y_i | P)$$

with the Banker *et al.* (1984) assumption of VRS.[10]

The model we want to estimate is the following: $\theta_i = z_i\beta + \varepsilon_i \geq 1$, where β are the parameters to be estimated and ε is the error term. However, as θ is not observable, we instead estimate:

$$\hat{\theta}_i = z_i\beta + \xi_i \geq 1 \tag{4.2}$$

where β are the parameters to be estimated and ξ is the error term and $\hat{\theta}$ were computed in the first stage. Clearly because $\hat{\theta}_i$ is derived from x_i and y_i, it will also be correlated with z_i. Therefore a bootstrap procedure to correct for serial correlation is necessary in the maximum likelihood estimation in the second stage. To implement the bootstrap, samples of pseudo-data x_i^*, y_i^*, z_i^*, are drawn from the density $\hat{f}(x, y, z)$. We follow Algorithm #1 with 2,000 replications as proposed by Simar and Wilson (2007).[11] The specific steps

[10] The definition of VRS in DEA can be found in the Appendix 4.A

[11] A detailed description of the algorithm can be found therein.

taken in the bootstrap algorithm can be found in Appendix section 4.C.

Because this bootstrap procedure was conceived for a left truncated dependent variable, our dependent variable is the inverse of the efficiency score as obtained by the input distance function. The bootstrapped values and original estimates are then used to construct estimated confidence intervals for the estimated parameters. These estimated parameters describe the influence of the environmental variables on the inefficiency scores obtained in the first stage.

4.3 Empirical efficiency analysis

4.3.1 Data and sample

We use cross sectional data for the year 2006 that pertain to the municipalities in the state of Saxony in Germany. In that year, the state had a total of 496 municipalities. About two-thirds (332) of all municipalities have a population of less than 5,000, and there are only three larger cities.[12] In total, 214,361 children were cared for in either public or non-profit child care facilities.

Our sample of municipalities is restricted in three steps. First, the municipalities with fewer than three individual child care providers are not disclosed in the statistic. Data on child care provision is only recorded for 282 municipalities (State Office of Statistics of Saxony, 2008b). Therefore, many small municipalities are disqualified from the analysis, but consequently, the remaining sample is more homogeneous. Second, observations that contain missing values in any of the potential inputs or outputs are excluded. Third, as non-parametric approaches are very sensitive to outliers, it is important that potential outliers are removed from the sample. Thus, an outlier detection procedure is applied to eliminate additional influential observations (Wilson, 1993). Ultimately, a sample of 213 municipalities remains.[13]

12 Leipzig with 506,578, Dresden with 504,795 and Chemnitz with 245,700 inhabitants.

13 Of the sample of 282 municipalities with more than three facilities, for 226 observations the data on all inputs and outputs is available. Among these observations 13 are found to be outliers and are subsequently dropped. Thus 213 municipalities

In child care, we distinguish between three distinct groups: the under 3-year-olds, the 3- to 6-year-olds and the 6- to 12-year-old participants in after school programs. In Saxony, the youngest of the three groups accounts for 13.8% of all children receiving care, the group of 3- to 6-year-olds forms the largest group with 47.5%, and 38.8% of children in child care are above the age of 6.[14] The differences in care intensity between the three groups are apparent in terms of the legally defined personnel-to-children ratio. For the youngest group, one care person may assume responsibility over no more than six children; for the middle group the ratio is 1:10 and 1:18 for the group of the oldest children. In other words, the number of personnel is dependent on the age structure of the children in care.

If a parent in Germany is interested in child care for his or her child, there are generally three different types of child care services to choose from (we call these Type I, II and III). Type I contains all non-profit, governmental (public) care centers. The non-profit, non-governmental centers define Type II (e.g., the welfare organizations of the evangelical and catholic churches). The municipalities subsidize the Type II providers for the number of children they care for. The subsidy is based on the average cost of care per child in the municipality's own (Type I) facilities. The final category, Type III, comprises private, for-profit child care centers. Type III facilities must cover their operating costs solely through tuition.[15] Most children who attend child care in Saxony do so either at a Type I (1,365 or 51.5%) or a Type II (1,257 or 47.5%) center. Thus, Type III centers play an insignificant role in the Saxon child care market. Because we are interested in municipal expenditure efficiency and because municipalities may organize the required number of child care centers either in fully public facilities (Type I) or through the

remain in the sample.

14 In the group of above 6-year-olds, the aggregate statistic at the state level includes cohorts up to the age of 12. However, from the population statistic at the municipal level, we are only able to distinguish the age group as 6- to 10-year-olds. As the number of 10 and 11-year-olds attending child care after school is relatively small, we use the population statistics on the 6- to 10-year-olds.

15 The attendance in fully private facilities is not recorded separately in the official statistics. However, most of these are located in one of the large agglomerations and are therefore excluded from our analysis.

non-profit providers (Type II), we analyze Type I and II facilities.[16]

Although we use a regionally restricted data set, our results may also be more generally applicable. Table 4.1 compares the child care situation across German regions in the following three dimensions: *facility density* (the number of facilities per 1,000 inhabitants), the *ratio of personnel to assigned places* and *assigned places per child* (the ratio of assigned places to the number of children in the relevant age group 1 to 12).

Table 4.1: Comparison of child care in Germany

	Facility density	**Personnel per assigned places**	**Assigned places per child**
Saxony	0.62	0.10	0.65
Eastern Germany	0.61	0.11	0.55
Western Germany	0.59	0.14	0.28
Germany	0.59	0.13	0.32

Sources: Federal Statistical Office (2011a,c).

Both the figures for facility density and the ratio of personnel to assigned places are very similar in Saxony and the other regions. The facility density is slightly higher in Saxony (0.62 compared to 0.59 for Germany in total), whereas the personnel ratio (0.10) is lower than the German average (0.13). However, with respect to the ratio of assigned places to children in the relevant age group, larger regional differences persist. Both the Saxon (0.65) as well as the eastern German ratios (0.55) are much higher than the one for western Germany (0.28). This reflects the lower availability of child care in the western parts of the country.

[16] Alternative issues that may influence the supply of or the demand for child care slots, such as rationing or the duration of daily care, do not require special consideration in our case. Rationing does not occur in eastern German municipalities, and most child care centers offer care for the whole day.

4.3.2 Quality Considerations

A challenge in efficiency analyses of the public sector is that output quality is difficult to control for because variables that describe quality are hard to define. The difficulty in controlling for quality could be seen as a potential shortcoming when considering such a sensitive area as child care services. Due to lack of data, the factors that actually drive the quality of a facility in a given form of child care (e.g., public child care) have so far only been investigated for child care providers in the USA (Blau, 1997; Blau and Hagy, 1998). These studies find a very weak influence of factors, such as education of the personnel or the ratio of personnel-to-children, on the human capital accumulation of the children. Instead, the socio-economic background of the parents determines the outcome of the children. Additionally, there is a considerable literature on the effects of early childhood programs on the children's future development.[17]

In Germany, the minimum personnel-to-children ratio is fixed by law, and the education of the personnel is comparable across the country. In addition, the Saxon law on child care services places further requirements on the advanced training of the personnel. The endowment of the facilities can be assumed to not vary much between facilities, as state law also defines the basic needs of facilities. All of these factors suggest that there are only slight quality differences between facilities and imply that the service quality is standardized within a narrow band. Consequently, quality differences are small in public child care between German regions (Felfe and Lalive, 2010). Finally, the regulations set minimum standards. It may therefore be possible

[17] Theoretical contributions find that investment in the young, and especially in socially disadvantaged children, is socially beneficial (Heckman and Masterov, 2007; Cunha and Heckman, 2007). Empirical findings support this theory. For instance, Andersson (1992) shows that the time of entrance into public child care has an impact on the child's future performance in school, and the timing of entrance, in turn, depends partly on the family's socio-economic background. Furthermore, it has been shown that high-quality public care outperforms informal care (Datta Gupta and Simonsen (2010b) for Danish pre-school programs and Fitzpatrick (2008) for pre-kindergarten programs in the USA). However, there are also studies that dismiss the advantages of public care relative to informal care (Datta Gupta and Simonsen, 2010a; Baker *et al.*, 2008).

that some municipalities invest more in child care than legally required (e.g., a better personnel-to-child ratio). However, because of the tight budget constraints that the municipalities face, this seems unrealistic to us.

To test whether quality differences in child care are present in our sample, we use data from the school readiness examination.[18] Before entering primary school, all children in Saxony are checked for their school readiness (regardless of attendance in public child care). The test measures the children's anthropometric measures as well as cognitive, non-cognitive, and motor skills as well as speech. We use the test scores on motor skills and speech as a proxy for the quality of care offered by the municipalities. In Saxony, 93% of children between three and six years old attend a child care facility (State Office of Statistics of Saxony, 2010). Most of these children attend child care for eight hours a day (i.e., full-time). Thus, it is plausible that child care influences the test outcomes. Nonetheless, these test scores are also influenced by other factors, such as the socio-economic background of the families. In the second stage analysis, we try to control for such factors with the average income in the municipalities and the unemployment rate as proxies for differences in the family background across the municipalities.

Table 4.2 shows the percentage of children with speech disorders and deficient motor skills of all children tested in the 29 Saxon counties in 2006. Each category may be divided into four classifications: no deficiency, minor deficiency, major deficiency and deficiency already in treatment. We then computed the ratio of children with one of the latter three classifications to all children for all counties. The figures in Table 4.2 present the corresponding average values for all counties in Saxony. We must resort to county-level data, as these highly sensitive data are not available at the level of municipalities. For all three groups (speech disorders as well as fine and gross motor skill deficiencies), the standard deviation is very low. This indicates that, of the Saxon children entering school in the different counties, a similar share has some deficiency in skills. We interpret this similarity as additional evidence for low variation in the quality of care. Despite the small differences in quality

[18] The data stems from the Saxon Ministry for Social Affairs that annually compiles information on the development of the children at school entry.

of care, we also calculated the efficiency scores using each of the three quality proxies as an additional output. The results differ only slightly from those without a quality proxy (see Appendix 4.D, Tables 4.11 and 4.12).

Table 4.2: Summary statistics of selected quality measures (county averages for the share of children with a deficiency of all children)

	Mean	St. dev.	Min.	Max.
Speech disorders	22.50	3.94	15.10	32.60
Fine motor skill deficiency	18.15	6.30	5.20	41.30
Gross motor skill deficiency	9.29	2.98	1.70	17.60

Source: Saxon Ministry for Social Affairs (2006)

4.3.3 Inputs and Outputs

In our specifications, we consider combinations of two different inputs and five different outputs (see Table 4.3). Expenditure variables that pertain to the provision of municipal child care services are plausible inputs. We only consider current expenditures (material costs and personnel); investment expenses do not enter the analysis. In some municipalities, the provision of child care is also partially provided by Type II (non-profit) organizations that receive transfers per child that they care for from the municipality. These transfers are added to the expenditures for materials recorded for the Type I (public) facilities. Together, these costs enter the analysis as the first input called "material expenditures."[19] Personnel expenditures are only recorded for the employees of the Type I centers. Therefore, we do not include the personnel expenditures in the analysis. Instead, the combined personnel (number of employed persons) in both Type I and Type II centers is included

[19] Although the funds transferred to the Type II centers are included in the material expenditures, we cannot know whether these funds are solely used for materials or if some are also used to pay personnel. However, as the transfer payments account for less than 5% of all material expenditures and as only 10% of the municipalities in our sample engage in transfer payments, the potential bias is marginal.

Table 4.3: Summary statistics of inputs and outputs

	Variable	Mean	St. dev.	Min.	Max.
Inputs					
Material expenditures (€)	(x_1)	218,037	193,119	1,584	1,308,953
Personnel (# persons)	(x_2)	41	32	9	171
Outputs					
Assigned places	(y_1)	397.50	297.70	103	1589
Facility density[a]	(y_2)	0.87	0.36	0.34	2.38
Weighted under 3 year olds[b]	(y_3)	72.29	59.84	11.03	336.36
Weighted 3-6 year olds	(y_4)	147.60	113.90	17.76	589.35
Weighted 6-10 year olds	(y_5)	133.75	98.52	23.52	549.47

[a] Facilities per 1,000 inhabitants

[b] Number of under 3 year olds times the county share of children under 3 receiving child care. The corresponding calculation was performed for the weighing of the other age groups.

as the second input. Because the wages of the child care employees in Type I (public) and Type II (non-profit) centers are regulated, the aggregation should not introduce a bias. The second input is thus ”personnel.” These inputs are summarized in the top section of Table 4.3.

The services a municipality offers are the outputs. De Witte and Geys (2011) discuss the appropriate choice of outputs for local public services and view the production as a two-stage process. The first stage determines the service potential and is described by the direct outputs produced with a given amount of inputs. The second stage is comprised of observable outcomes that are potentially influenced by demand for the service: thus, the level of these outputs does not directly reflect the production process (e.g., number of children receiving care). A direct output variable is therefore the total number of assigned places (i.e., the legally allowed capacity aggregated for all age groups in a given municipality).[20] We call this variable “assigned places”. The second output we consider is “facility density” which is measured

[20] Data on the number of assigned places disaggregated by the age of the child is not available. Therefore, the potential output variable is limited to the total number of assigned places.

as the number of facilities per 1,000 residents. Because the municipalities can control how many facilities they operate, this variable accounts for size differences and differences in fixed costs from having multiple units. The above two outputs describe the service potential and are considered direct outputs.

However, using only these two outputs may mask the differences in care intensity between the age groups. Because in the year 2006 only children above the age of three until school age (6-year-olds) had a legal claim to a place in a child care facility, this group forms the largest group for whom services need to be produced. Two additional fractions remain, namely those above the age of 6 who receive after school care, and those below the age of 3. On the one hand, children under the age of 3 require significantly more supervision than older children and therefore care for this relatively small group is still costly and should also enter the analysis as an output. On the other hand, school age children above the age of 6 form the third large group of children receiving care in public child care facilities. Of all 1- to 3-year-olds 45.4% were in child care in 2006, of 3- to 6-year-olds 92.6% and of 6 to 10-year-olds 67.2%.

To better account for the differences in the three age groups, we construct a weighted output proxy. The county-specific shares of children in each of the three age groups receiving care can be used to construct a demand index.[21] The shares are multiplied by the number of children in each age group in a given municipality. These three indices, weighted under 3 year olds, weighted 3- to 6-year-olds and weighted 6- to 10-year-olds, are then used as outputs in two alternative specifications.[22] The summary statistics of the potential outputs are found in the bottom half of Table 4.3.

The DEA is conducted for four different specifications. The four models that result from the different input-output combinations are summarized in

[21] We must resort to the figures at the county level because the corresponding data are not made available for the municipalities.

[22] These are partially demand driven and thus do not just reflect the municipal service potential or productive efficiency. However, to test whether the age structure has an influence on the efficiency, we resort to these alternative outputs. The findings show that differences are small, and therefore, our preferred model for the second stage regression excludes these outputs.

Table 4.4. All models contain the two inputs, "materials" and "personnel", but contain different output combinations. Model A only contains one output, namely the number of "assigned places". Model B has "facility density" as an additional output. The "weighted number of children" in the three relevant age groups constitute the outputs in Models C. In addition to these three outputs, the "facility density" is the fourth output in Model D.

Table 4.4: Overview of input and output combinations in the models

	Model A	**Model B**	**Model C**	**Model D**
Inputs				
x_1	×	×	×	×
x_2	×	×	×	×
Outputs				
y_1	×	×		
y_2		×		×
y_3			×	×
y_4			×	×
y_5			×	×

4.3.4 Results of the efficiency analysis

Municipalities have a set of resources (inputs) at their disposal with which they produce the required services (i.e., outputs). Different input-output relations might stem from three sources: (i) different input prices, (ii) additional inputs that are used productively and (iii) additional inputs that are used unproductively. Different input prices are unlikely because of heavy regulation and fixed state wage scales. We can also exclude (ii) as a source of more excessive input use, as the discussion in section 4.3.2 has shown. The DEA specification that we use implies that the deviation from the efficient frontier stems from inefficient use of inputs. In other words, in inefficient municipalities, the same level of output of the same quality could be produced with fewer inputs. Unobservable differences that influence efficiency may be present (such as unobserved inputs/outputs/quality differences or measurement errors in the variables); however, we cannot identify these.

With the described inputs and outputs, we evaluate four different DEA specifications (models A, B, C and D). For all four models, we compute the CRS as well as the VRS efficiency scores in both the input and output orientations. Furthermore, we test the level of scale efficiency using the CRS specification. Before selecting a specific model for the second stage regression we discuss the results of the efficiency analysis in this section.

Table 4.5 shows the results from the four different specifications in the input orientation with VRS. The median efficiency of the four specifications ranges from about 72% to 75%. This means that in the median municipality the same level of output could be achieved using 28% to 25% fewer inputs. In Models A and B, the minimum efficiency is 45%, and in Models C and D is 36%. The standard deviation of the efficiency scores ranges from 13.8 to 15.3. Of the 213 municipalities in our sample, Models A and C contain the same number of efficient units, 20, whereas Models C and D deem 26 units fully efficient. Therefore, the percentage of efficient municipalities lies between 9% and 12% of the sample. The inclusion of the facility density as an input has a slightly larger influence on the efficiency than the disaggregation of the outputs with respect to the three different age groups. Considering the range of output proxies, the variation between the models is relatively small. The results are robust across specifications.

Table 4.5: Results of the efficiency analysis: VRS input orientation

Model	Min.	Median	St. dev.	Efficient	% efficient
Model A	0.449	0.717	0.138	20	9%
Model B	0.449	0.731	0.141	26	12%
Model C	0.357	0.730	0.153	20	9%
Model D	0.357	0.752	0.152	26	12%

Table 4.6 contains the results of the same four models in the output orientation with VRS. The differences in the results from the input orientation are only slight. In Model C the minimum efficiency is lower than in the input orientation of the respective model. This indicates that there is more variation with three outputs than with one. With more differentiated outputs

there may be larger divergence from the frontier in the extreme. Models B and D that contain the variable “facility density” have slightly less deviation. The median efficiencies of all models, in both orientations, lie within 11% points.

Table 4.6: Results of the efficiency analysis: VRS output orientation

Model	Min.	Median	St. dev.	Efficient	% efficient
Model A	0.465	0.749	0.136	20	9%
Model B	0.519	0.782	0.118	26	12%
Model C	0.261	0.758	0.150	20	9%
Model D	0.484	0.830	0.113	26	12%

As it is not immediately clear whether the municipalities may choose to lower their outputs or to increase their inputs, the production process may be either input or output oriented. If the efficiencies of CRS and VRS are equivalent, then the orientation is irrelevant and the municipalities can be said to be operating on an efficient scale. To test scale efficiency, we compute for each unit the ratio of its CRS efficiency score to the VRS score, as defined by equation (4.3):

$$SE_i = \frac{CRS_i}{VRS_i}. \tag{4.3}$$

When the CRS and VRS scores are equal, a municipality is fully scale efficient and receives a score of 1. If, however, a municipality deviates from the efficient frontier under either of the orientations, then either increasing or decreasing returns to scale are present. A municipality may then be deemed relatively more or less efficient depending on the respective orientation.

On average, the municipalities in our sample are from 89% to 92% scale efficient and thus operate close to constant returns to scale. As the economic interpretation of adjusting inputs to meet a specified level of output is more probable in this case, we proceed in the second stage with the efficiency scores from input orientation.

Clearly, there are only small differences in the minimum and median efficiencies between the different specifications. We use the Spearman rank correlation to test the correlation of the rank of individual observations in the different specifications (see Table 4.7). Models A and B render almost identical rankings; these models receive a correlation of 0.97. Models C and D are also very similar; the two specifications have a rank correlation of 0.95. The correlation between the remaining models is slightly lower (0.63-0.68). Nonetheless, all four models are highly positively correlated.

Table 4.7: Rank correlations between the four models in the input orientation

Models	A	B	C	D
A	1			
B	0.97	1		
C	0.68	0.63	1	
D	0.66	0.68	0.95	1

For the ensuing regression analysis, the efficiency scores from Model B (computed in the input orientation) are used as the dependent variable. This model contains two inputs and two outputs that capture the most important factors with respect to efficiency.[23]

4.4 Second stage regression

In the previous section, we have shown that there are differences in the technical efficiency of child care provision. We now proceed to explain these differences systematically, making use of variables in three broad categories: (1) variables that describe the political economy, (2) variables that describe the local demography and (3) a variable that accounts for the fact that there may be competition among facilities of Types I and II within a municipality.

[23] As shown in the previous section, the alternative models deliver very similar rankings. Therefore, in specifications using one of the other three sets of efficiency scores as the dependent variable, the results are also similar.

4.4.1 Explanatory variables

In the category of political economy, we consider the influence of the share of open-ended grants to own tax income, the status of the mayor (full-time salaried or uncompensated) and a Herfindahl index of political concentration on the efficiency of child care provision.

Open-ended grants are transfers from the Free State of Saxony to the municipalities. Such grants compensate for the heterogeneous economic situation across the municipalities and aim to achieve fiscal equalization within the state. The higher the own tax revenues per capita, the lower the amount in transfers a municipality receives from the government. For estimation purposes, we put the grants in relation to tax revenues. Thus, the grant shares are comparable across municipalities of different size. Because there are no restrictions on these grants with regard to investment decisions, it is possible that financial aid is not fully used to adjust expenditures to local needs (e.g., to adapt local expenditures to changes in demand, such as demographic change). The effect of such grants on municipal expenditure inefficiency is analyzed by Kalb (2009). He finds that in the provision of county roads in the state of Baden-Württemberg in southern Germany, higher intergovernmental grants typically lead to higher inefficiency (flypaper effect).[24] We test the influence of grants as stated in our first hypothesis:

Hypothesis 1: A larger share of grants increases inefficiency.

The second political variable is a dummy variable that is zero if a municipality has a full-time salaried mayor. Municipalities are required by law to have a full-time salaried mayor if their population exceeds 5,000. If the population is smaller, the municipality is free to choose between a salaried and an uncompensated mayor. In our sample, 48% of the municipalities have fewer than 5,000 inhabitants. Therefore, smaller municipalities are encouraged to form administrative collectives, by which they may share a full-time salaried mayor. Economic literature discusses the influence of the status of

[24] Similar results are obtained by Silkman and Young (1982) for the United States and De Borger and Kerstens (1996) for Belgian municipalities.

a municipality's mayor on expenditures (Stumm and Corrigan, 1998; Deno and Mehay, 1987). Full-time salaried mayors may be more qualified (e.g., have degrees in business administration or experience in politics). Thus, municipalities with a full-time salaried mayor are expected to be more efficient.

Hypothesis 2: Having an uncompensated mayor increases inefficiency.

The third political variable is a Herfindahl index of political fragmentation. The index H is constructed as the sum of parties' squared share of votes (p):[25]

$$H = \sum_{i=1}^{I} p_i^2. \tag{4.4}$$

A high index value is indicative of strong leadership. A high political concentration implies low political fragmentation. High fragmentation has been shown to increase expenditures and deficits, whereas more concentration leads to lower spending (Ashworth *et al.*, 2005; Roubini and Sachs, 1989; Roubini *et al.*, 1989).

Hypothesis 3: More political concentration decreases inefficiency.

Further factors that are likely to influence the efficiency of child care provision are the ongoing demographic changes. Almost all municipalities in Saxony already face a declining population, and this development will continue during the next decades (State Office of Statistics of Saxony, 2008a). Nevertheless, there is again strong heterogeneity: in the period between 2005 and 2020, some municipalities will lose about one quarter of their population, whereas others will remain almost constant.

Usually, larger municipalities (in terms of inhabitants) can be expected to be more efficient for two reasons. First, due to increasing professionalism within organizations, when demand for specific services is sufficiently high, the employees may be more specialized in a narrow field (e.g., care givers for specific age groups instead of for all age groups). Second, a larger population can act as an insurance against changes in demand. A larger municipality with more public services may be more flexible in the allocation

[25] Independent candidates are collected in one category.

of its resources. For example, if one child care center is faced with decreasing demand, a facility in another part of town may need additional personnel.

Hypothesis 4: A larger population decreases inefficiency.

To capture an additional demographic aspect, we also use the share of persons over 65 years old in relation to the total population in 2006. This is both a demographic and a political economy variable. An increasing share of the elderly implies that the median-voter gets older and thus, interest in financing child care facilities tends to decrease. Therefore, less funding is made available to finance child care services, and the efficiency of child care provision is expected to increase (Epple and Romano, 1996).[26]

Hypothesis 5: A larger share of over 65-year-olds decreases inefficiency.

Finally, we use a variable to analyze whether there is competition in the provision of child care between Type I and Type II facilities. We make use of a dummy variable indicating whether both types are present within a municipality. When Type II facilities are present the willingness to pay for Type I facilities may be lower. Thus through competition the presence of external service providers could provide an incentive for the municipality to operate more efficiently.[27]

Hypothesis 6: The presence of transfers to Type II providers decreases inefficiency.

Table 4.8 summarizes the above variables and their expected influence on child care inefficiency. Table 4.9 contains the summary statistics of the explanatory variables used in the second stage regression.[28]

[26] Simultaneously, a large share of elderly implies a smaller influence of younger cohorts (potential parents) on political outcomes. Altruism towards grandchildren could counteract this effect.

[27] However other studies have shown that competition from private schools does not necessarily increase the technical efficiency of public schools (Grosskopf *et al.*, 2001). When both types of child care facilities are present (and thus transfers from Type I to Type II facilities are made), administrative costs may accrue, which lowers efficiency. Additionally, when parents have a choice between different types of facilities, the facilities may have an incentive to spend inefficient amounts to remain attractive.

[28] In Appendix 4.E Table 4.13 presents the pairwise correlation among all independent

Table 4.8: Variable description and expected influence on inefficiency

Category	Variable description	Expected influence
Political economy	Ratio of grants to tax income	+
	Uncompensated mayor dummy	+
	Party concentration	-
Demography	Total population (in 1,000)	-
	Share in over 65 year olds	-
Competition	Type II dummy	-

Table 4.9: Summary statistics of environmental variables

Variable description	Variable	Mean	St. Dev.
Ratio of grants to tax income	(z_1)	1.24	0.73
Uncompensated mayor dummy	(z_2)	0.29	0.45
Political concentration	(z_3)	0.39	0.09
Total population (in 1,000)	(z_5)	7.19	5.63
Share of over 65 year olds	(z_4)	0.22	0.03
Type II dummy	(z_6)	0.16	0.37

4.4.2 Estimation results

Table 4.10 shows the coefficients of the truncated regression. The bootstrapped standard errors appear below the coefficients in parentheses. The dependent variable is the efficiency score of Model B computed in section 4.3.4. Because the inverse of the efficiency scores was used as the dependent variable, a negative sign reduces inefficiency (or increases efficiency).[29]

The coefficients of an uncompensated mayor, the share of over 65-year-

variables and shows that correlation between the variables is not a serious problem in our data set.

[29] Appendix 4.F shows the regression results using the efficiency scores obtained from efficiency analyses with the quality proxies as an additional output. The results are similar across all specifications, especially with regard to the sign of the coefficients and the level of significance.

Table 4.10: Truncated regression results

Variable	Model B
Ratio of grants to tax income	-0.035
	(0.028)
Uncompensated mayor dummy	0.085*
	(0.049)
Political concentration	0.052
	(0.222)
Total population (in 1,000)	-0.022**
	(0.006)
Share of over 65 year olds	2.504**
	(0.768)
Type II dummy	0.001
	(0.060)
Constant	1.001**
	(0.195)
$\hat{\sigma}_{\xi}$	0.239**
	(0.015)
Log-likelihood	29.681

Note: N=193. Bootstrap corrected standard errors in parentheses.
** (*) denotes a 5% (10%) level of significance.

olds and the total population are statistically significant. All else held equal, having an uncompensated mayor increases inefficiency in the provision of child care services by 8.5% compared to having a full-time salaried mayor. Although the state encourages cooperation among smaller municipalities to support them in installing a professional mayor, many municipalities still retain their uncompensated mayors.

Our hypothesis that municipalities with more elderly are more efficient in the provision of child care is not corroborated. A larger share of the elderly influences the inefficiency positively. For a given level of output, excessive inputs (expenditures on materials and personnel) are used. Aging municipalities may have an incentive to have inefficiently high public spending for the young when the municipalities engage in fiscal competition over the more mobile younger population group (Montén and Thum, 2010). When this is the case, a large share of the elderly can have a negative impact on the efficiency of child care provision.

The size of the total population has a negative influence on inefficiency (i.e., larger municipalities are more efficient). An increase of 1,000 inhabitants reduces inefficiency by 2.1%. This reflects that larger municipalities may benefit from more professional structures and general scale effects. Additionally, they may be more flexible in shifting capacities to meet local needs.

In light of the insignificant coefficient on the share of grants, we find no support for the flypaper effect. The size of open-ended grants a municipality receives does not influence the efficiency of child care provision.[30] We interpret this result positively, as municipalities would rather adjust the supply of child care than channel additional resources from state grants to cover over-sized provision. The insignificance of the Type II dummy indicates that municipalities that transfer funds to Type II (non-profit) providers use the given resources equally efficiently as municipalities that provide the service internally. Furthermore, we do not find a statistically significant effect of the remaining political variable.

4.5 Discussion and policy implications

We have analyzed the efficiency of municipalities in the provision of child care services. Specifically, we used data on the municipalities in the Free State of Saxony in Germany. We employed a two-stage analysis in which we first computed the efficiency scores using the non-parametric DEA method, and in a second stage, we regressed the efficiency scores in a truncated regression. From a policy perspective, it is reasonable to assume that the production process follows an input orientation. The expenditures are adjusted to the number of offered places, and not vice versa.

In the first stage, we identified differences in efficiency in the provision of

[30] The same line of argument could be applied to an alternative variable *per capita income* that we used in the alternative specification (see Appendix 4.F). A negative influence on efficiency could be expected if higher incomes translate via higher parental fees into higher spending. Again, we do not find a significant influence of this variable.

public child care services. The median municipality has efficiency reserves of up to 25%. For the least efficient municipalities, we find efficiency reserves of over 50%. Different model specifications render similar efficiency scores, which supports the choice of the inputs and outputs.

The regression analysis in the second stage shows that professionalism in administration is an important factor in explaining differences in efficiency. This aspect is particularly relevant in a state where many small municipalities are struggling with adverse demographic developments and dwindling resources. By encouraging cooperation and professionalizing administration, more appropriate expenditure decisions can be made at the local level. This finding is further corroborated by the fact that municipalities with a larger population tend to be more efficient. A larger population encourages more professional administration. Higher efficiency could thus be achieved by merging municipalities. With regards to municipal amalgamations, our results imply that child care is not an area in which per capita cost savings can be achieved through increased service provision (high-scale efficiency). Even if the production process follows constant returns to scale, more professional administration may increase efficiency and reduce costs.

The demographic composition also influences efficiency. We find that municipalities with a larger share of over 65-year-olds over-spend on family friendliness. To remain attractive to families with children, municipalities with aging populations have an incentive to spend more on child care. A generous provision of child care is important in the location choice of families, particularly for those in which both parents are active in the labor force.

In the future, an investigation aimed at the facility level could complement our analysis. Although the municipalities are in charge of financing this service, a closer consideration of the facilities in selected municipalities could render more evidence with respect to competition and scale effects. Facilities in selected municipalities could be surveyed to conduct such a detailed evaluation.

Appendix

Appendix 4.A Derivation of the DEA models

4.A.1 Constant and variable returns to scale efficiency (CRS and VRS)

The original DEA estimator was proposed by Charnes, Cooper and Rhodes (1978) and yields the constant returns to scale efficiency scores for the input orientation.[31] Subsequently the model was extended by Banker, Charnes and Cooper (1984) to incorporate the case of variable returns to scale. In either case, the efficiency of any decision making unit (DMU) is measured from the maximization of a ratio of weighted outputs with respect to weighted inputs, subject to the restriction that the ratios for the rest of similar DMU's are less than or equal to unity. Charnes *et al.* (1978) emphasize the use of the nomenclature DMU to distinguish that entities other than firms or industries can also be studied, including non-profit organizations for which weighing by market prices is not possible.

Given a set of N DMU's, using M inputs and generating S outputs. Then for the i-th DMU the known inputs and outputs are represented by the vectors x_i and y_i respectively. For all N units the input matrix X has the dimensions $(M \times N)$ and the output matrix Y is represented by an $(S \times N)$ matrix. By weighing the inputs and outputs used by the units in the set, a frontier which envelopes all units is constructed. The restriction dictates that no unit may lie above the frontier. The frontier is constructed by specifying optimal weights of outputs and inputs by considering the ratios of all outputs and inputs used by each DMU. The ratio is defined as $(u'y_i/v'x_i) \leq 1$, where u is an $(M \times 1)$ vector of output weights and v is an $(K \times 1)$ vector of input weights. By maximizing the constrained efficiency measure the weights of inputs and outputs for each unit are determined. Additionally the constraint $v'x_i = 1$ is imposed to limit the number of solutions, and the weights

31 For a textbook examination of the DEA models used here, see for instance Fried *et al.* (2008)

are changed from u and v into μ and ν. The multiplier form of the linear programming problem is subsequently written as:

$$\begin{aligned} \max_{\mu,\nu} \quad & (\mu' y_i), \\ \text{s.t.} \quad & \nu_i = 1, \\ & \mu' y_j - \nu' x_j \leq 0, \quad j = 1, 2, ..., N, \\ & \mu, \nu \geq 0 \end{aligned} \tag{4.5}$$

the above problem involves $N + 1$ constraints. Therefore, the equivalent envelopment form of the problem is preferred when solving since it uses fewer constraints, namely $S + M$:

$$\begin{aligned} \min_{\theta,\lambda} \quad & \theta, \\ \text{s.t.} \quad & -y_i + Y\lambda \geq 0, \\ & \theta x_i - X\lambda \geq 0 \\ & \lambda \geq 0. \end{aligned} \tag{4.6}$$

θ is a scalar and is the efficiency score of the i-th unit. Therefore the linear programming problem needs to be solved N times to obtain the scores for each DMU. $\theta \leq 1$ where a value 1 indicates a technically efficient unit that lies on the frontier. λ is a $(N \times 1)$ vector of constraints.

This problem describes the envelopment form of the input oriented constant returns to scale model. To account for variable returns to scale, an additional convexity constraint on λ, namely $N1'\lambda = 1$ is needed (Banker *et al.*, 1984). Where N1 is an $(N \times 1)$ vector of ones. Through the incorporation of the variable returns to scale (VRS) assumption, the model becomes the following:

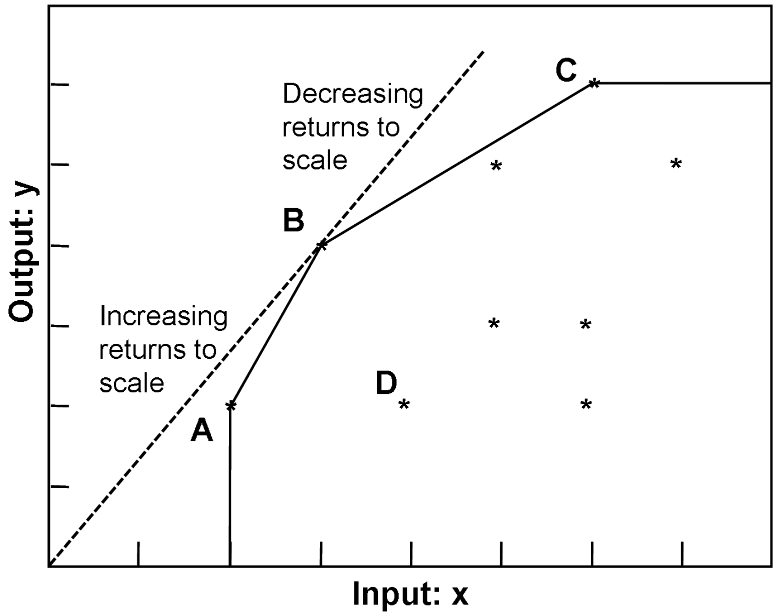
$$\begin{aligned} \min_{\theta,\lambda} \quad & \theta, \\ \text{s.t.} \quad & -y_i + Y\lambda \geq 0, \\ & \theta x_i - X\lambda \geq 0 \\ & N1'\lambda = 1 \\ & \lambda \geq 0. \end{aligned} \tag{4.7}$$

The problem above describes the envelopment form of the input oriented variable returns to scale model according to Banker *et al.* (1984). Again θ is a scalar efficiency index and is interpreted as how much the inputs could be reduced while still achieving a given level of output in a particular unit. When θ is equal to one, for a given level of output no reduction in inputs is possible. In this case the given DMU lies on the frontier and is a best-practice unit. For inefficient units, θ is strictly positive and indicates the neighboring (reference) units, to which an inefficient unit is to be compared. The λ therefore guides how the linear combinations of the efficient units for the efficiency frontier are formed. A unit with $\lambda = 1$, is its own reference point and must therefore lie on the frontier, and consequently also has an efficiency index $\theta = 1$. On the other hand values of $\lambda > 0$ indicate by how much a given reference unit contributes to the virtual unit on the frontier to which an inefficient unit is to be compared. The linear program is solved for each DMU separately, and hence the most favorable weights for each will be assigned individually. Inefficiency in each case is the radial distance of θ from the frontier, and since the frontier takes a value of 1, the inefficiency is a value (1-θ). The value is then a percentage of how much inputs could be reduced in a given unit without reducing output.

Appendix 4.B Returns to scale

Figure 4.1 depicts the CRS and VRS frontiers for the one input one output case. The input x is on the horizontal axis and the output y is on the vertical axis. The solid line forms the variable returns to scale (VRS) frontier, according to which units A, B, and C are efficient, and D is inefficient. The additional points depict additional inefficient units. The dashed line depicts the constant returns to scale (CRS) frontier which is a ray from the origin, and only deems one unit, namely B as fully efficient. The increasing and decreasing returns to scale are therefore computed as the the differences between the two frontiers. Along the ray "below" unit B (not including point B) units can realize increasing returns to scale, whereas units operating "above" unit B (not including point B) in terms of output operate under decreasing returns to scale.

Figure 4.1: Returns to scale in production for the one input one output case



Appendix 4.C Bootstrap algorithm

Algorithm #1 (Simar and Wilson, 2007)

1. Using original data compute $\hat{\theta}$.

2. Use the method of maximum likelihood to obtain an estimate of $\hat{\beta}$ of β as well as an estimate $\hat{\sigma_\varepsilon}$, of the true variance, σ_ε, in the truncated regression of $\hat{\theta}$ on z_i in $\hat{\theta}_i = z_i\beta + \xi_i \geq 1$ using the $m < n$ observations where $\hat{\theta} > 1$.

3. Loop over the next three steps L times to obtain a set of bootstrap estimates.

   3.1 For each $i = 1, ..., m$ draw ε_i from $N(0, \hat{\sigma_\varepsilon^2})$ distribution with left truncation at $(1 - z_i\beta)$.

3.2 Again for each $i = 1, ..., m$ compute $\hat{\theta}_i^* = z_i\hat{\beta} + \varepsilon_i$.

3.3 Use the method of maximum likelihood to estimate the truncated regression of $\hat{\theta}_i^*$ on z_i, yielding $(\hat{\beta}^*, \hat{\sigma_\varepsilon^2})$.

4. Use the bootstrap values and the original estimates $\hat{\beta}$, $\hat{\sigma_\varepsilon}$ to construct estimated confidence intervals for each element of β and for σ_ε as described below.

Since $\hat{\theta}_i$ is a consistent estimator of θ_i, maximum likelihood in step 2 will lead to consistent estimates of β. Step 3 is a parametric bootstrap of a nonlinear regression model.

The number of bootstrap replications used to construct estimates of confidence intervals in the algorithm is defined by L. We use the recommended $L = 2000$.

Appendix 4.D Model B and specifications accounting for quality

Table 4.11: Efficiency results of Model B including additional quality variables

Model	Min.	Median	St. dev.	Efficient	% efficient
Model B	0.449	0.731	0.141	26	12%
Model B-speech	0.449	0.770	0.146	37	17%
Model B-fine	0.449	0.760	0.142	32	15%
Model B-gross	0.449	0.757	0.142	33	15%

Table 4.12: Spearman rank correlation of efficiency scores between Model B and specifications additionally accounting for quality

	Model B	Model B-speech	Model B-fine	Model B-gross
Model B	1			
Model B-speech	0.81	1		
Model B-fine	0.87	0.84	1	
Model B-gross	0.86	0.93	0.94	1

Appendix 4.E Correlation between environmental variables

Table 4.13: Correlation matrix for the environmental variables

	z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8
z_1	1.000							
z_2	0.084	1.000						
z_3	-0.051	0.266	1.000					
z_4	0.129	-0.171	-0.225	1.000				
z_5	0.028	-0.387	-0.324	0.409	1.000			
z_6	-0.010	-0.147	-0.140	0.144	0.213	1.000		
z_7	-0.230	-0.125	-0.065	-0.495	-0.067	-0.078	1.000	
z_8	0.155	-0.227	-0.243	0.415	0.526	0.192	-0.584	1.000

Note: Variables z_7 and z_8 are *average per capita income in a municipality* and *unemployment rate*, respectively. These variables are used in an alternative regression (see Appendix 4.A).

Appendix 4.F Alternative estimation results

Table 4.14: Alternative truncated regression results with efficiency scores from Model B and quality adjusted efficiency scores as dependent variable

Variable	(1)	(2)	(3)	(4)	(5)	(6)
Ratio of grants to	-0.039	-0.035	-0.035	-0.037	-0.037	-0.036
tax income	(0.029)	(0.027)	(0.028)	(0.033)	(0.032)	(0.031)
Uncompensated	0.076	0.085*	0.085*	0.138**	0.050	0.093*
mayor dummy	(0.049)	(0.046)	(0.048)	(0.054)	(0.054)	(0.052)
Political	-0.033	0.052	0.052	-0.074	0.078	0.028
concentration	(0.230)	(0.226)	(0.220)	(0.251)	(0.252)	(0.249)
Share of over 65 year	2.096**	2.504**	2.504**	2.718**	2.778**	2.708**
olds	(0.976)	(0.758)	(0.766)	(0.893)	(0.899)	(0.842)
Total population (in	-0.019	-0.022**	-0.022**	-0.016**	-0.017**	-0.0195**
1,000)	(0.006)	(0.006)	(0.006)	(0.007)	(0.006)	(0.006)
Transfers dummy	-0.005	0.001	0.001	0.022	0.029	0.038
	(0.061)	(0.053)	(0.060)	(0.069)	(0.066)	(0.065)
Per capita income	2.12E-05					
	(3.33E-05)					
Unemployment rate	-0.210					
	(0.494)					
Constant	1.322	1.001	1.001	0.882	0.850	0.893
	(0.550)	(0.194)	(0.194)	(0.210)	(0.235)	(0.214)
$\hat{\sigma}_\xi$	0.238	0.239	0.238	0.249	0.245	0.239
	(0.015)	(0.016)	(0.015)	(0.019)	(0.018)	(0.018)
Log-likelihood	29.053	29.681	29.681^{+}	32.924	33.335	35.504
Observations	N=187	N=189	N=189	N=176	N=181	N=180

Column (1), alternative specification with dependent variable from Model B. (2) naive regression of Model B, no bootstrap. (3) Model B, no bootstrap but robust standard errors. (4) Model B with *speech disorders* as an output quality proxy. (5) Model B with *deficiency in fine motor skills* as an output quality proxy. (6) Model B with *deficiency in gross motor skills* as an output quality proxy.

Note: ** (*) denotes a 5% (10%) level of significance. Bootstrap corrected standard errors in parentheses except in model (2) where no bootstrap procedure was implemented and model (3) where robust standard errors are shown. $^{+}$denotes the log-pseudolikelihood.

Chapter 5

Conclusion

This thesis considered the influence of demographic developments on publicly provided goods at the local level. The three main chapters analyzed distinct aspects of public goods provision and local population change. The model presented in Chapter 2 focused on ageing whereas the model in Chapter 3 considers shrinking. The empirical analysis in Chapter 4 suggests that the demographic structure may have a decisive impact on expenditure decisions and hence also on the efficiency with which municipal services are provided.

As was stated in Chapter 1, it is clear that the ongoing demographic developments will have wide reaching consequences for individuals and societies. However, broad sweeping judgments that aging and shrinking populations will cause stunted economic growth and lead to a melt down of welfare states are imprudent. Instead a careful and detailed analysis of repercussions for specific issues is more conducive. The focus here laid on the consequences for the provision of publicly provided goods in municipalities. Some of the results are counter-intuitive to what would be expected if populations were growing, or if the perspective were of the national level.

Chapter 2 focused on the competition between municipalities, when the number of young residents is declining. It is shown that in "young" gerontocracies the provision of the publicly provided good for the young is provided at an inefficiently low scale. However, when aging advances the competition for young residents can intensify to such an extent that the provision actually becomes excessive (with respect to the level that would be welfare maximizing). In this setting the threat of ageing towns is not the exploitation of the smaller young cohorts but rather the excessive incentives for municipalities to remain attractive for the mobile young residents.

The potential inability of local governments to scale back investments to reflect the changing demand of the population is further analyzed in Chapter 3. The model focuses on the effects of a shrinking population (i.e. a smaller subsequent generation) on the provision of intergenerational publicly provided goods. In comparison to cases of population growth or stability, the choices of private consumption and public goods provision is different when the population shrinks. Generally the successive generation benefits from the investments it inherits from the previous generation. However, it may be optimal from the view of a small ensuing generation not to provide any additional public goods at all. Furthermore, population loss lowers the inefficiency of the intergenerational spillover. Only if no costs are associated with the upkeep of the inherited infrastructure will the ensuing generation surely benefit. If the successive generation is smaller and has to allocate resources toward the upkeep of the inherited stock, then the inheritance may burden the budget to the point of insolvency. Therefore accounting for costs of upkeep will result in a lower provision of public goods than would be the case if such costs we ignored.

The consequences of both of the above formal analyses may have a compounding effect. If excessive investments in publicly provided goods that are intergenerational in character are made today to attract young mobile residents, then the future (inevitably) smaller generations, will face infrastructures that exceed their demand and which maintenance will excessively burden their budgets. Therefore awareness among local policy makers with regard to the long term consequences of their decisions is needed. Given the current population trends, increasing polarization between communities will arise. Inevitably, since the population in vast regions will decline, some communities will lose. However, when the consequences are recognized and policy is adapted, a smaller population does not need to equate to a lesser quality of life for the remaining residents.

The empirical analysis in Chapter 4 provided additional insights into the provision of child care; a local public expenditure category that is highly sensitive to demographic change. Using data from the Free State of Saxony, in eastern Germany, the analysis identifies significant differences in the efficiency of the provision between municipalities. In the median municipality the same level of child care could be achieved with 25% less resources if the municipality were operating efficiently. Thus the number of children receiving care could be increased significantly given the existing resources. This finding is particularly relevant for the current effort of the federal government to guarantee a place of care also for all 1 and 2-year-olds in the country.

Furthermore it was shown that the professionalism of the local administration and the age structure of the municipality, influence the efficiency with which child care services are provided. Municipalities that have a professional mayor tend to be significantly more efficient. Moreover, municipalities with a relatively large share of elderly (persons above the age 65), tend to be less efficient in the provision of child care services. This finding seems to corroborate the proposition raised in Chapter 2, that in fiscal competition for the mobile young, communities with a large share of elderly may attempt to attract young people by offering an inefficiently generous bundle of public goods.

It is beyond the scope of this thesis to evaluate specific alternatives on how public actors can adapt to the demographic developments. However, one practicable option is the implementation of systematic evaluation of demographic risk. Whenever local public governments plan projects with long lasting impacts the (infrastructure) project is also to be evaluated with respect to demographic sustainability. In fact, the Free State of Saxony recently adopted a comprehensive "demographic test" to be conducted for all investment decisions with long lasting impacts (Saxon State Chancellery, 2011). The aim is to prevent mis-investments both at the state and local levels and thus reduce the fiscal risk. Specific investment decisions and in particular new infrastructure investments, must be evaluated with respect to the local demographic development in terms of aging and shrinking. Furthermore, should demand dwindle the cost of downscaling are also to be accounted for when new investments are proposed.

This thesis was focused on the small spatial scale to highlight the fact that policy implications for the national level may not necessarily hold true for local communities. Furthermore, it was shown that the impact of demographic change is also very differentiated across different spatial types and locations.

Populations are continuously changing, however the current trend in many developed countries has a foreseeable trajectory toward populations with increase median age. Shrinking will also begin in ever more regions. Even if birth rates would suddenly peak above replacement rate, due to population momentum, the trend toward ageing and smaller populations is inevitable in a growing number of countries. As national populations undergo this change the local populations may become more polarized. This thesis has only touched upon very specific aspects of population change in the very specific context of the welfare state. Nonetheless as mentioned in the introductory notes to this thesis, the spatial scale and realms of influence of demographic change go beyond economics and touch on wide reaching aspects of our daily lives.

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